Formulation and Evaluation of New Control Structures in the Great Lakes System

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1 Introduction

1.1 Background

Great Lakes – St. Lawrence water levels are critically important to the Canadian and US economies. Extreme and persistent high or low water levels can have negative impacts on the ecosystem and result in millions of dollars of losses to a variety of stakeholder groups such as hydropower producers, the shipping industry, shoreline property owners and recreational boaters. Although the system is currently managed by control structures at two locations (Lake Superior and Lake Ontario outflows are controlled), there is concern that future extreme climates will generate water supply sequences to the system that will negatively impact the system performance and substantially increase the frequency and persistence of extreme water levels (see for example Angel and Kunkel (2010)).

At the direction of the International Joint Commission (IJC), the International Great Lakes Study (IUGLS) Board is investigating possible changes to the present method of regulating the outflows of Lake Superior to better meet the contemporary needs of the interests and also taking into consideration the potential effects of climate change. The Board, as part of their first Phase completed in December 2009, examined physical processes and possible ongoing changes in the St. Clair River and their impacts on levels of Lake Michigan-Huron. In that effort, the Study Board recognized the importance of climate change, noting the world-wide consensus among scientists that climate change, driven by increasing concentrations of greenhouse gases in the atmosphere, is occurring and will continue, though its effects will differ from one region to another. In their first report to the IJC, the Study Board recommended “…that the need for mitigating measures in the St. Clair River be examined as part of the comprehensive assessment of the future effects of climate change on water supplies in the Upper Great Lakes basin in Report 2 of the Study, on Lake Superior regulation” (International Upper Great Lakes Study Board, 2009).

Recognizing that the water levels and outflows of the Great Lakes are constantly changing in response to the hydroclimatic, hydraulic and meteorological conditions of the basin, the Study Board realized that it would be difficult to design a regulation plan now that would be optimal for all future conditions including climate change. Moreover, it became apparent early in the Study process that Superior regulation alone could do little to reduce the risk posed by extreme water levels downstream of Lake Superior.

With climate change as the driving force, and recognizing the significance of future uncertainties and the attendant inability to craft an Upper Great Lakes regulation plan that could reduce risks from extreme water levels, the Board embraced the notion of Adaptive Management as a means to cope with- and respond to- the inherent uncertainties associated with the system as more is learned over time. Recognizing this was stretching outside the original Plan of Study, which included studying climate change as it relates to the existing regulation structure at Sault St. Marie, but did not provide generally for considering climate change implications or actions
beyond Superior regulation, the Board raised this with the IJC at an October 2009 meeting. The IJC responded in a letter to the Board in April 2010, after consulting with governments, giving their approval for the Study to conduct the examination of climate change impacts on water levels, and indicated it should “include consideration of a full range of options available to all potentially affected sectors across the Great Lakes – St. Lawrence River system at an exploratory level.” Furthermore, guidance from the two governments indicated this exploratory analysis was to include both structural and non-structural measures, and be conducted within the existing Study budget and timeframe.

1.2 Purpose and Scope

As a result of the IUGLS Study Board’s phase 1 recommendations to consider mitigation in the St. Clair River in light of climate change, and the additional guidance in April 2010 from the IJC to examine options to address climate change at an exploratory level, a multi-lake regulation task was added to the Study as part of the Adaptive Management Group work plan. The purpose of this effort would be to provide an exploratory conceptual analysis of how new control structures in the system could be used to try to minimize the risks posed by extreme water levels outside of the historic range. This required a new plan formulation effort to develop multi-lake regulation operating rules, requiring close coordination with the IUGLS Lake Superior Plan Formulation and Evaluation Group. The plan formulation for multi-lake regulation was to consider new hypothetical structures in the St. Clair and/or Niagara rivers that, in combination with the St Marys River and St. Lawrence control structures (or some revised structures), and possible further mitigating structures downstream of Montreal, would provide a variety of ways to manage levels throughout the Great Lakes that meet a set of hydrologic objectives aimed at minimizing potential risk of extreme water levels. The effect on Lake Ontario and the St Lawrence River levels would also be considered, or could be specified as constraints. The new alternatives were to be run with a number of water supply scenarios from a suite of stochastic and climate change water supply scenarios with an eye to at least determining how much a multi-lake regulation plan could mitigate extreme levels throughout the system, and perhaps for other objectives, such as moderating levels so they were more like historic levels.

Given that the Study did not want to formulate a multi-lake regulation plan that simply passed damages downstream, the ability to assess the implications for Lake Ontario and the St. Lawrence River was considered critical to the analysis. This however, would limit the number of water supply scenarios available for assessment, as only the water supply scenarios from the Lake Ontario-St. Lawrence River Study (LOSLR) (International Lake Ontario-St. Lawrence River Study Board, 2006) could be utilized, since these are the only water supply scenarios available to the IUGLS for the entire Great Lakes basin that include the Ottawa River flow. This was not considered a major concern, as these scenarios provide numerous extreme conditions to test. Again, as noted in the IJC letter, this would all be done at an exploratory level and would not include detailed benefit/cost assessments. However, the evaluation would make use, to the extent practical, of existing evaluation models and performance indicators developed for the Lake Superior Plan Formulation and Evaluation component of the IUGLS study. Some
preliminary effort would be taken to examine past literature to determine the types of structures and excavation that have been proposed in the past along with any costs estimates updated to current dollar values. No new effort to design structures would be considered. This report is not intended to recommend a new Great Lakes control strategy for implementation. Instead, this report is designed to inform the IJC about the range of possible water level management improvements and the corresponding estimated costs for two additional control structures on the Great Lakes. This report details the methods and results of the IJC requested exploratory multi-lake regulation study.

1.3 Reservoir Optimization

Although there are many previous studies on Great Lakes management and multi-lake regulation, most of them were motivated by the occurrence of high water levels while this study was motivated by recent low water levels that occurred over the last ten years. Perhaps the most extensive previous study of multi-lake regulation on the Great Lakes was the IJC Levels Reference Study (Levels Reference Study Board, 1993).

As part of the mandate of the Levels Reference Study, several different initiatives were studied and reported by Task Group 1 of the Working Committee 3 (Task Group 1, Working Committee 3, 1993). The Task Group commissioned an optimization study in order to develop and test a range of system-wide regulation options that included two (lakes Superior and Ontario), three (lakes Superior, Erie and Ontario) and five (Superior, Michigan-Huron, Erie and Ontario) lake regulation. The optimization method was based on the flow network algorithm with a perfect knowledge of net basin supplies for the ninety-year period considered. The objective was based on reducing the cost for the overall system using constraints defined by the six stakeholder interests that were integrated into a weighted single interest satisfaction curve. For example, the optimized results showed a compression of water levels on all lakes (Lake Superior about 9 cm, Lake Michigan-Huron about 52 cm, Lake Erie about 81 cm and Lake Ontario by about 57 cm). The range of water levels, however, at Montreal showed a small increase of 13 cm. The excursions of water levels above and below the critical thresholds, defined by the Citizen Advisory Committee, were also reduced in all lakes while showing an increase at Montreal.

The Levels Reference Study concluded that additional control structures were not economically justified and thus the Study Board did not pursue recommending the construction of any physical works. In fact, the Study Board recommended physical works not be looked at any further. Fletcher and Ponnambalam (1998) followed up the work done in the Levels Reference Study and present an improved methodology and then demonstrate it using a simplified case study on the Great Lakes. Another important study on the Great Lakes system is the Lake Ontario – St. Lawrence study (International Lake Ontario-St. Lawrence River Study Board, 2006) that focuses on improving the current Lake Ontario regulation plan.

A variety of other studies have also optimized the operation of the existing control points on the Great Lakes. Seifi and Hipel (2001) applied two stage stochastic linear programming with
recourse to the Great Lakes System (Lake Superior through Lake Ontario considered). Seifi and Hipel (2001) formulated an optimization model that yields an optimal annual set of monthly release targets for the existing control points (Lake Superior and Lake Ontario) across multiple water supply scenarios. Their objective was to minimize the sum of absolute deviations from regulatory storage and release targets and the nonlinear routing equations for connecting channel flows were approximated using a first-order Taylor series approximation.

There is a rich history of reservoir optimization modelling and it is beyond the scope of this study to review it. Labadie (2004) provides a thorough critical review of reservoir optimization modeling and the possible range of approaches. Very recently water resource system and reservoir optimization studies have started to tackle the question of optimal reservoir management under future climate change projections. Examples include the work of Vicuna et al. (2010), Lee et al. (2009) and Lee et al. (In press).

The specific goal of this multi-lake regulation study is to determine to what extent the base case management system levels can be improved when additional control structures are added on the Upper Great Lakes. In particular, this study focuses on developing multi-lake regulation strategies to mitigate extreme climates, or more specifically, extreme water supply scenarios. The additional control structures and channel changes considered are in the St. Clair River to control the outflow of Lake Michigan-Huron (MH) and in the Niagara River to control the outflow of Lake Erie.

Our approach to multi-lake optimization was to develop a set of multi-lake rule curves defining the target releases for each control point. We selected to first formulate the mathematical structure of these rule curves and then optimize the rule curve coefficients directly via global optimization methods instead of the more traditional approach of formulating an optimization model to optimize releases directly and then fitting releases via some sort of multivariable regression analysis. The main benefit of our approach is that it provides a maximum amount of flexibility in terms of being able to directly simulate for any set of control points the non-linear routing and water levels of the Great Lakes – St. Lawrence system (instead of approximating these values) and in terms of the objective functions utilized. The other obvious benefit is that the optimization result is a release policy that can be simulated for any other NBS sequence. As noted by Labadie (2004), inferring release rules from an optimal sequence of releases by regression analysis can fail and the alternative of developing release rules based on optimal releases from other methods can require extensive trial and error experimentation and can have little general applicability. A variety of previous studies have solved the reservoir optimization problem with an approach consistent with the general approach to optimize rule curves utilized in this study. Some example studies directly optimizing rule curve coefficients or rule curve structures with single or multi-objective heuristic global optimization algorithms include Oliveira and Loucks (1997), Chen et al. (2007), Valeriano et al. (2010), Afshar et al. (2011) and Li et al. (2010).
1.4 Outline of Report

The remainder of this report is structured as follows. Section 2 describes the details of the simulation-optimization methodology utilized to develop optimized multi-lake rule curves. This includes a description of the three different regulation and hydrologic routing simulation models invoked during the optimization process, development of the primary optimization objective, as well as the secondary objective and the constraints in the optimization formulation, and highlights how multiple water supply sequences were incorporated into the optimization formulation. Results are described in Section 3 and all of these compare various new UW plans using either two or four control points to the base case performance of the current regulation strategy (two control points). Results in Section 3.1 demonstrate the system performance under the current or base case regulation strategy. Section 3.2 details single objective optimization results for the primary management objective (keeping lake levels within historical simulated extremes) and compares the University of Waterloo (UW) plans with and without the lower St. Lawrence River locations in the formulation. Section 3.3 describes the multi-objective optimization results or tradeoff between the primary objective (keeping lake levels within historical extremes) and the secondary objective (minimizing structural + dredging costs associated with optimized releases). Section 4 lists the conclusions and suggestions for future work.
2 Simulation-Optimization Methodology

The first step in this multi-reservoir optimization study was to define the simulation models used to calculate lake levels and lake outflows across the system. These simulation models varied depending on what multi-lake regulation policy is to be simulated. A total of three different simulation models (each of which is a combination of a variety of simulation codes) were developed in this study and these are described in detail in Sections 2.1, 2.2 and 2.3. These three simulation models are then utilized in a variety of ways to assess system performance of multiple multi-lake regulation plans as defined by multi-lake rule curves that are described in Section 2.4. This assessment can be conducted within or independent of the optimization models formulated to optimize the management objectives and constraints described in Sections 2.5.1 through 2.5.5.

2.1 Base case regulation and hydrologic routing model

The Great Lakes – St. Lawrence system with the current regulation strategies is called the base case. From Lake Superior down to the Niagara River, the base case is simulated with the Coordinated Great Lakes Routing and Regulation Model (CGLRRM) that was originally developed by the National Oceanographic and Atmospheric Administration’s (NOAA’s) Great Lakes Environmental Research Laboratory (GLERL), in conjunction with the US Army Corps of Engineers (USACE) and Environment Canada (EC). The origin of CGLRRM was through work by Quinn (1978) and refined later by Clites and Lee (1998). Today, CGLRRM is sanctioned by all relevant agencies from Canada and the US that make up the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. Also regulation Plan 58DD is used to simulate Lake Ontario regulation in the base case. The base case simulation is further extended to the St. Lawrence system with various empirical equations. This simulation approach is consistent with the simulation approach used in the Lake Ontario St. Lawrence (LOSL) study (LOSL, 2006).

In this study, the base case simulation model described below and all other simulation models developed (see Sections 2.2 and 2.3) are driven by all or portions of a data set of 50,000 years of stochastic inflows and routing factors that were initially produced for the LOSL (2006) – or simply LOSL (2006) in the rest of this report. This data set is typically referred to in this report as the LOSL 50,000 year stochastic NBS data set or the 50,000 year LOSL NBS sequence. Fagherazzi et al. (2005) report details of how the 50,000 year NBS sequence was generated. The LOSL 50,000 year stochastic NBS sequence was requested by the LOSL Study Board to act as a hydrologic test series for evaluation of alternative Lake Ontario Regulation plans. Such a test series avoided the obvious shortcomings of simulating regulation plans under the very limited historical period NBS data set. The following description of this data set, available from the LOSL study website (http://www.losl.org/boardroom/50kyear_stochastic_e.php) overviews the data set:

“This large sample of possible future hydrologic conditions also proved necessary to properly estimate the time value of erosive losses. “Stochastically” generated here means that a computer model was developed to produce a 50,000-year sequence of supplies to each of the
Great Lakes, the Ottawa River and other downstream tributary flows, based on the statistical characteristics of the twentieth century supplies. The stochastic hydrology model included the important probabilistic relationships between the supplies from one year to the next, their seasonal patterns and their quarter-month to quarter-month correlations. The stochastic model also preserved other important statistical properties of the system, such as the varying length of drought periods and high supply periods, and the cross-correlation of supplies among basins (i.e. the probability that wet or dry conditions would occur in the various drainage basins at the same time). Each of these characteristics of the hydrology has a natural random component. This randomness is also captured in the stochastic model so that the 50,000-year generated sample contains a distribution of possible hydrology composed of mostly typical supplies with the appropriate number of rare and extremely rare events needed to fully test the regulation plans.”

As noted in the LOSL (2006) study, this 50,000 year stochastic NBS sequence is best for assessing the average performance of various regulation plans. In order to utilize this data set to develop new regulation plans under extreme future climates, it was decided in conjunction with the Study Board, to seek out unique and extreme 70 to 80 year slices of this data set that could be thought of as representative of possible future extreme NBS sequences resulting from climate change. This identification procedure is described further in Section 2.5.3 and Appendix 3.

**Simulation of Lake Superior**
CGLRRM uses Plan 77A to regulate Lake Superior and calculates the outflow and surface elevation of the lake at the end of each time step. Lake Superior is simulated on an hourly time step in CGLRRM and the regulation time step is monthly (along with NBS and diversions).

**Simulation of Mid-Lakes (Lake MH down to Niagara River)**
CGLRRM uses a hydrologic routing model of the Great Lakes and is comprised of continuity equations for each lake, solved using a second-order finite-difference technique (Quinn, 1978) with a numerical time step of one hour. The small numerical time step here provides the most accurate results and is computationally feasible given the base case regulation strategy is simulated only when the NBS scenario changes. It uses as inputs time series of net basin supply (NBS) and any diversions, as well as initial outflows and lake levels, for each lake. The model calculates discharge from each lake from an open-water lake-to-lake stage-fall-discharge equation with regression-based coefficients. In the winter, flows in the connecting channels are naturally reduced by the presence of ice. CGLRRM does not explicitly simulate the ice retardation process and instead this phenomenon must be described by a time series of ice retardation rates (e.g., reduction in connecting channel flow in m³/s or cms) for each channel. Caldwell (2008) provides a detailed description of the ice retardation approach in CGLRRM and assesses the long term impacts of apparent roughness factors, which include ice retardation on lake levels.

CGLRRM simulates the surface elevation of Lakes Michigan-Huron (considered to be a single lake, MH, for hydraulic purposes), St. Clair and Erie and uses as input the flows of the St. Marys
River. The Chicago and Welland Canal diversions are also inputs to the model (coordinated monthly averages). Most inputs for CGLRRM in all analyses in this report are specified on a monthly time step for Lake Superior outflow (St. Marys River coordinated monthly averages flow), residual NBS of Lake MH and St. Clair, ice and weed retardation of St. Clair and Detroit Rivers. CGLRRM inputs for residual NBS of Lake Erie and ice and weed retardation of Niagara River were specified at a quarter monthly time step. Also the Chicago diversion is constant (91 cms) and the Welland Canal diversion is monthly constant (220, 211, 222, 247, 244, 246, 248, 239, 244, 243, 243, 237 cms) for the whole simulation period. Outputs of CGLRRM assessed are the monthly mean lake levels, beginning-of-month levels, and monthly flow rates for the connecting channels.

**Simulation of Lake Ontario**

In the base case, Lake Ontario levels are simulated by the current Lake Ontario regulation plan (Plan 58DD) that controls outflows from the lake and is simulated based on a compiled program provided by Environment Canada. LOSL (2006) provides more details about this regulation plan. The simulation time step in Plan 58DD is daily and the regulation time step is quarter monthly. Simulation inputs are also quarter-monthly.

**St. Lawrence River System Simulation**

Simulation of the St. Lawrence River system is consistent with the simulation approach used in the LOSL study (LOSL, 2006). This part of the simulation uses quarter monthly inputs and simulates with a numerical time step that is also quarter monthly. The hydraulic behavior of the St. Lawrence River system is extremely complex and thus the empirical approach used here represents a substantial simplification of the system. As such, simulation of this part of the system is less accurate than that of Great Lakes system.

St. Lawrence system simulation has two parts. In the first part, surface elevations at various locations along the St. Lawrence system down to Saunders Headwater are calculated based on empirical equations. Among these locations, Iroquois Headwater Level and Saunders Headwater Level are required for the optimization. The following empirical equations show how to estimate St. Lawrence River elevations (in m) at Iroquois Headwater (IHW) and the Saunders Headwater (SHW) evaluation points:

\[
Kingston = \text{Lake Ontario mean level} - 0.03 \text{ m} \quad (1)
\]

\[
IHW = Kingston - N_{IHW} \left[ \frac{Q}{(23.6495 \times (Kingston - 62.40)^{2.2886})} \right]^{(1/0.4158)} \quad (2)
\]

\[
SHW = Kingston - N_{SHW} \left[ \frac{Q}{(21.603 \times (Kingston - 62.40)^{2.2586})} \right]^{(1/0.3749)} \quad (3)
\]

Where \( N \) is the apparent roughness factors that vary quarter-monthly due to ice, weeds, winds etc. and is given for the 50,000 years of stochastic data from the LOSL study and \( Q \) is the Lake Ontario outflow in cms which is calculated by the rule curve of Lake Ontario. It should be noted that when SHW level are computed greater than 73.87 m from the above equations, the SHW
level is set to a value of 73.78 m. This rule is based on how the system is operated currently to avoid high levels at SHW.

In the lower St. Lawrence, downstream of Saunders Headwater, the St. Lawrence system from Lac St. Louis and downstream to Jetty 1 in Montreal is simulated based on empirical equations used in the LOSL study (LOSL, 2006). From this part of the simulation, only the level of Lac St. Louis at Pointe Claire and the level in Montreal harbour (at Jetty 1) are required in the optimization. Both of these are calculated based various flows and other factors. The spatial locations of the flows and levels are depicted in Figure 1 and the full simulation logic is included (as Matlab syntax) in Appendix 1. Only two of the equations to calculate water levels are described below.

![Figure 1- The map of the Lower St. Lawrence River system being simulated in the study.](image)

The Lac St. Louis at Pointe Claire level is calculated by the following equation:

$$H_{PCL} = 16.57 + \left( F_{STL} \times \frac{Q_{STL}}{604.0} \right)^{0.580}$$  \hspace{1cm} (4)

where $F_{STL}$ is the dimensionless ice factor for Lac St. Louis and $Q_{STL}$ is outflow from Lac St. Louis in cms. A 50,000 year time series of stochastic $F_{PCL}$ factors is available from the LOSL study.

The following empirical equation also from the LOSL study is used to calculate Montreal harbour level (at Jetty 1):
\[
H_{MTL} = [K_{MTL}(0.001757 Q_{STL} + 0.000684 Q_{DPMI} + 0.001161 Q_{STFRN} + 0.000483 Q_{STMAU})]^{0.6589} + 0.9392T
\]  

(5)

where \( K_{MTL} \) and \( T \) are dimensionless ice-weed factors and tidal signal for Montreal harbour respectively. \( Q_{STL} \) is Lake St. Louis outflow, \( Q_{DPMI} \) is outflow of the Des Prairies plus the Mille Iles, \( Q_{STFRN} \) is outflow of the Saint-Francois River at Hemming Falls, \( Q_{STMAU} \) is the outflow of the St. Maurice River at La Gabelle all in cms. Time series of 50,000 years of stochastic data (from the LOSL study) includes all these required input parameters and input flows (\( K_{MTL}, T, Q_{STFRN}, Q_{STMAU}, Q_{CHAT}, Q_{CARL} \)) to calculate the unknown flows (\( Q_{STL}, Q_{DPMI}, Q_{VAUD}, Q_{STA} \)) and ultimately the level at Montreal harbour. Appendix 1 details how these other flows are computed.

2.2 Two-point Multi-lake regulation and daily hydrologic routing model

As a part of this study, the current regulation of Lake Superior and Lake Ontario is replaced with new rule curves for these locations. In other words, the same set of existing structures is assumed to exist but the only change to the system is to optimize new operating rules at these locations. The two-point multi-lake regulation and daily hydrologic routing model simulates levels under such a regulation framework. Simulation of the system with two control points (one on Lake Superior and one on Lake Ontario) is coded in MATLAB based on the hydrologic routing component of CGLRRM and a mass balance for Lakes Superior and Ontario. This section describes the simulation of this system and its differences to the base case described in Section 2.1. Similar to the base case, this simulation is also extended to the St. Lawrence system with the same empirical equations.

**Simulation of Lake Superior**

Outflow of Lake Superior respects the regulation rules coded in CGLRRM that split the lake outflow between the side channels (for hydropower) and gates (flow through the rapids). Therefore the outflow of Lake Superior assigned by the rule curve (or any release policy) is always adjusted to respect the limits associated with the side channel and gate split. Knowing the initial surface elevation of Lake Superior, the actual release amount and the NBS for the current time step, the mass balance equation is applied to calculate the surface elevation of Lake Superior at the end of each time interval (daily time step in our simulation).

**Simulation of Mid-Lakes (Lake MH down to Niagara River)**

Simulation of this part of the system is not affected by the modifications in the regulation of Lake Superior and Ontario compared to the base case. Therefore, it calculates discharge from Lake MH, St. Clair and Erie from open-water lake-to-lake stage-fall-discharge equations (the same equations as the base case simulation model). This results in a system of three linear equations with three unknowns that are surface elevation changes of Lakes MH, St. Clair and Erie for each numerical time step. This system of equations is solved iteratively in the base case (by CGLRRM with an hourly time step). Note that in our two-point simulation model, the matrix solution approach is utilized since it is much more computationally efficient (this simulation model must be simulated for any new 2-point regulation plan generated during the optimization.
procedure). Moreover, in the base case, the numerical time step of one hour is used for the most accurate results; while here, a numerical time step of one day is used to make the simulation more computationally efficient. Experiments showed this efficiency gain comes at very little change in numerical accuracy (e.g. average monthly lake levels changing by only a few cm at worst).

Just like the base case simulation model description in Section 2.1, the two-point simulation model uses as inputs an NBS data set, diversions, as well as initial outflows and lake levels for each lake, and a time series of ice retardation rates (e.g., reduction in connecting channel flow in cms) for the St. Clair, Detroit and Niagara Rivers.

The Chicago and Welland Canal diversions are also inputs to the model (coordinated monthly averages). Most inputs for simulation of Great Lakes system in all analyses in this report are specified on a monthly time step for residual NBS of Lakes Superior, MH and St. Clair, ice and weed retardation of SCR and Detroit River. Inputs for residual NBS of Lake Erie and ice and weed retardation of Niagara River were specified at a quarter monthly time step. Also the Chicago diversion is constant (91 cms) and the Welland Canal diversion is monthly constant (220, 211, 222, 247, 244, 246, 248, 239, 244, 243, 243, 237 cms) for the whole simulation period. Outputs of the simulation model include monthly mean lake levels, beginning-of-month levels, and monthly flow rates for the connecting channels.

Simulation of Lake Ontario
There are some physical/structural limits that Lake Ontario outflow must respect. These limits are computed based on the Lake Ontario regulation plan (Plan 58DD) provided by Environment Canada. The release value (target release) obtained from the Lake Ontario rule curve is modified (actual release) to respect the limits of Lake Ontario releases. The actual releases from Lake Ontario are governed by the target release as specified by a rule curve/release policy and the Lake Ontario release constraints. Quarter-monthly releases from Lake Ontario must satisfy a complex suite of physical or structural constraint limits explained in Fay (Annex 3, International Lake Ontario St. Lawrence River Study, 2005). For example, these constraints include respecting maximum week to week flow changes and limiting flows during ice formation periods. Knowing the initial surface elevation of Lake Ontario, the actual release amount from Lake Ontario and the NBS for the current time step, the mass balance equation is applied to calculate the surface elevation of Lake Ontario at the end of each time numerical time step (daily in our simulation).

St. Lawrence System Simulation
Simulation of this part of the system is not affected by the modified regulation of Lakes Superior and Ontario; therefore, this part of the system is simulated exactly the same as in the base case.

2.3 Four-point Multi-lake regulation daily hydrologic routing model
In order to have more flexibility for the multi-lake regulation, two new control points are to be considered at the outlet of Lakes MH and the outlet of Lake Erie. Hence, the Hydrologic routing
part of the simulation model should be modified to only simulate Lake St. Clair and Lake Erie surface elevations and St. Clair River flow. Here, surface elevation of Lakes Superior and MH are calculated by mass balance. This section explains the changes to the 2-point simulation model of the system relative to Section 2.2 that are required when the system is controlled instead with four control points.

**Simulation of Lake Michigan-Huron**
The rule curve of Lake MH determines its outflow. Therefore, all inflows and outflows of the lake are known and mass balance is applied to calculate the end of interval (daily time step) Lake MH surface elevation.

**Modified Hydrologic Routing Model for the System with Four Control Points**
The rule curve of Lake MH and Erie define the outflows of these two lakes and the surface elevation of Lake MH. Therefore, the Hydrologic routing model should be modified to calculate only surface elevations of Lake St. Clair and Lake Erie and the Detroit River flow. Hence the system of linear equations with two unknowns must be solved here to calculate the changes in the surface elevation of Lakes St. Clair and Erie and lake levels at the end of a numerical time step. The system of two linear equations is reported in the Appendix 1 of the report. The numerical time step is daily.

**Some Concerns in Simulation of the System with Four Control Points**
When the rule curve defines the outflow of Lake MH, the open-water lake-to-lake stage-fall-discharge equation between Lake MH and St. Clair does not necessarily result in the St. Clair River flow which is determined by the rule curve. This suggests that the conveyance capacity of St. Clair River must be changed. This issue will be addressed in Section 2.5.5 where an estimate of the cost of structural channel changes to the system is estimated. Another concern is that, in some extreme situations, Lake MH outflow is so high that it decreases the surface elevation of Lake MH and increases the surface elevation of Lake St. Clair so that surface elevation of Lake St. Clair becomes higher than that of Lake MH. The simulation model in this study is not able to handle this issue and will fail. One way to avoid this situation is to reduce the outflow of Lake MH in the corresponding time step. In this study, the outflow of Lake MH is reduced to zero in the corresponding numerical time step (day). Similar situations may occur between Lake St. Clair and Lake Erie. One way to resolve or patch this issue is to reduce the flow of corresponding connecting channel to zero. However, if the connecting channel flow in such situations is already small, this strategy may not always resolve this issue. Note that none of the optimized solutions evaluated later in this report, exhibited behavior requiring our strategy above to be implemented.

2.4 Multi-Lake Rule Curve Development
In this study, various regulation plans, which are all referred to as “UW plan” hereafter, consisting of a set of rule curves for operation of the entire Great Lakes-St. Lawrence System are developed. The UW plans reported on here are designed to regulate the two existing control
structures downstream of Lake Superior and Lake Ontario (Moses Saunders Dam) as well as two hypothetical (potential) control structures downstream of Lake Michigan-Huron and Lake Erie. Later work will develop plans for only two or three control points. All physical/hydraulic constraints existing in Lake Superior and Lake Ontario control structures are considered and respected in the UW plans.

2.4.1 Form of Rule Curves

Four rule curves were generated that respond to beginning of period water level and flow conditions throughout the system to make release decisions corresponding to the four control points (Superior, Michigan-Huron, Erie, and Ontario). Release decisions are made starting at Superior and then sequentially moving downstream. As such, for example, the Superior planned release at time $t$ is known when the release for time $t$ from Michigan-Huron is being determined. The regulation time step is quarter monthly (a new release is computed after each quarter month) at all control points (compared to the daily numerical routing time step) except for Lake Superior where the regulation time step is monthly to be consistent with the current regulation time step in Plan77A. The rule curve target releases are defined by up to three additive components quantifying the storage condition upstream and downstream of a control point as well as a baseline flow term. The exact form of the rule curve at control point $i$ ($i=1, 2, 3,$ and $4$) is given as follows:

$$\text{Target Release}(i, t) = \text{Component 1}(i, t) + \text{Component 2}(i, t) + \text{Component 3}(i, t) + \text{Baseline Flow}(i)$$

(6)

where component 1 is a function of the upstream storage condition of control point $i$, component 2 is a function of the storage condition downstream of control point $i$ down to the next control point, $i+1$, and component 3 is a function of the storage condition between the next two control points, $i+1$ and $i+2$, and the baseline flow from lake $i$ is the outflow of a lake when all the three components (upstream and downstream) are at normal conditions (i.e., storage everywhere is normal, no excess or shortage of water anywhere). This baseline flow is a parameter of the rule curve equations that is optimized. For convenience, all these components and the baseline flows were defined to be unit-less numbers that when combined to a unit-less target release. The unit-less target release calculated from equation (1) is transformed into units of m$^3$/s as described in Section 2.5.4 for each lake. Components 1, 2, and 3 are explained in the following subsections and as detailed below, the target releases specified by the multi-lake rule curves are governed by 39 parameters for a single season (Section 2.4.2 details how we consider two seasons and thus a total of 78 rule curve parameters to be optimized with four control points). Appendix 2 lists all 39 decision variables/rule curve parameters for a season along with their associated allowable ranges.

**Component 1:**

Component 1 is a piecewise linear function of the upstream storage condition with three break points as shown in Figure 2. This function is defined by six rule curve parameters. The x-axis
represents the water storage condition upstream of control point \( i \) \((i=1, \ldots, 4 \) for Superior, MH, Erie, and Ontario\) which can be calculated as:

\[
\text{UpstreamStorage}(1) = \frac{A_{\text{Sup}} (Z_{\text{Sup}} - \text{avg}Z_{\text{Sup}})}{n_{u,\text{SP}}} \tag{7}
\]

\[
\text{UpstreamStorage}(2) = \frac{[[R_{\text{Sup}} - \text{avg}R_{\text{Sup}}] + A_{\text{MH}} (Z_{\text{MH}} - \text{avg}Z_{\text{MH}})]}{n_{u,\text{MH}}} \tag{8}
\]

\[
\text{UpstreamStorage}(3) = \frac{[[R_{\text{MH}} - \text{avg}R_{\text{MH}}] + A_{\text{SC}} (Z_{\text{SC}} - \text{avg}Z_{\text{SC}}) + A_{\text{ER}} (Z_{\text{Er}} - \text{avg}Z_{\text{Er}})]}{n_{u,\text{ER}}} \tag{9}
\]

\[
\text{UpstreamStorage}(4) = \frac{[[R_{\text{Er}} - \text{avg}R_{\text{Er}}] + A_{\text{ON}} (Z_{\text{ON}} - \text{avg}Z_{\text{ON}})]}{n_{u,\text{ON}}} \tag{10}
\]

where \( A_{\text{Sup}}, A_{\text{MH}}, A_{\text{ER}}, \) and \( A_{\text{ON}} \) are the lake area of Superior, Michigan-Huron, Erie, and Ontario, respectively. \( Z_{\text{Sup}}, Z_{\text{MH}}, Z_{\text{SC}}, Z_{\text{ER}}, \) and \( Z_{\text{ON}} \) are the lake levels at the beginning of the time interval. \( \text{avg}Z_{\text{Sup}}, \text{avg}Z_{\text{MH}}, \text{avg}Z_{\text{SC}}, \text{avg}Z_{\text{Er}}, \) and \( \text{avg}Z_{\text{ON}} \) are the historical average monthly lake levels. \( R_{\text{Sup}}, R_{\text{MH}}, \) and \( R_{\text{Er}} \), are the release volume planned to be released from Superior, Michigan-Huron, and Erie during the next time interval, and \( \text{avg}R_{\text{Sup}}, \text{avg}R_{\text{MH}}, \) and \( \text{avg}R_{\text{Er}} \) are the historical average monthly flow volume in that interval. The variables \( n_{u,\text{SP}}, n_{u,\text{MH}}, n_{u,\text{ER}}, \) and \( n_{u,\text{ON}} \) are normalizing constants, and are equal to 3.9901e+010, 1.3595e+011, 8.6788e+009, and 2.7662e+010, respectively. The normalizing constants are utilized to generate unitless upstream storage conditions so that both axes on Figure 2 are unitless. The normalizing constants were calculated via a trial and error procedure based on the statistics of the simulated historical period storage terms in each of Equations 7-10 (e.g., volume of Superior releases and volume of excess/shortage of storage on Lake MH for Equation 8). Note that the values of these normalizing constants are not critical since the optimization algorithms will adjust the six rule curve parameters to work well for these fixed values of the constants.

![Figure 2- Component 1 representing the contribution of upstream storage condition on regulated release](image)

According to Equations 2-5, upstream storage condition in Lake Superior is only a function of current Superior lake level, but in the other control points, the planned release from the upper
control point is also considered in the upstream storage condition confirming that release decisions made upstream sequentially affect the release decisions downstream.

Upstream storage of zero suggests that the control point is in normal condition (i.e., no excess or shortage of water). Any positive value for upstream storage suggest that the control point is at the excess water condition (i.e., water budget available is above than average historical records), and a negative number for upstream condition suggest that there is a shortage of water at the control point.

Component 1 consists of four segments: high shortage, low shortage, low excess, and high excess, resulting from three break points. Component 1 responds differently (different associated slopes in the piecewise linear function) for different upstream storage conditions. As shown in Figure 2, a, b, c, and d are the slope values associated to high excess, low excess, low shortage, high shortage cases, respectively. The component 1 slope values are positive following the direct relationship between upstream water storage and release. In other words, the higher the water storage is, the higher the release is. The following assumptions were also considered for determination of these slopes: b and c are supposed to have relatively close values; high excess segment is supposed to have a considerably steeper slope than the low excess segment (\(a \gg b\)), and high shortage segment is supposed to have a considerably steeper slope than the low shortage segment (\(d \gg c\)). When the upstream storage is in high excess or high shortage cases, it means that the lake level is well above or well below the normal level and it is going (or has gone) beyond the desirable lake level range. As such, the justification of \(a\) and \(d\) having considerably steeper values is to avoid going beyond the desirable range as component 1 has a direct linear contribution in determining the target release (see equation 1).

The center breakpoint is located at upstream storage of zero (normal condition). As shown in Figure 2, the breakpoints specifying the high shortage and high excess conditions are determined through location parameters \(e\) and \(f\). Overall, component 1 has six parameters, including four slope parameters \(a\), \(b\), \(c\), and \(d\) as well as two location parameters \(e\) and \(f\), to be determined through optimization.

Component 2:

Component 2 is a two-segment linear function of the storage condition immediately downstream of a control point down to the next control point as shown in Figure 3 and it is defined by two rule curve parameters. For control point \(i\) (\(i=1, 2, 3,\) and \(4\)), the x-axis is the current water budget available between control point \(i\) and control point \(i+1\) (next control point) being calculated through the following equations:

\[
\text{DownstreamStorage}_{im}(1) = \left( \frac{A_{MH}(Z_{MH} - \text{avg}Z_{MH})}{n_{d,MH}} \right)  
\]

\[
\text{DownstreamStorage}_{im}(2) = \left[ \frac{A_{SC}(Z_{SC} - \text{avg}Z_{SC}) + A_{ER}(Z_{Er} - \text{avg}Z_{Er})}{n_{d,ER}} \right]  
\]

\[
\text{DownstreamStorage}_{im}(3) = \left[ \frac{A_{ON}(Z_{ON} - \text{avg}Z_{On})}{n_{d,ON}} \right]  
\]
\[
\text{DownstreamStorage\_im}(4) = \frac{(Z_{\text{Jetty1}} - \text{avg}Z_{\text{Jetty1}})}{n_{d,\text{Jetty1}}} \tag{14}
\]

where \(Z_{\text{Jetty1}}\) is the water level at Jetty1 station and \(\text{avg}Z_{\text{Jetty1}}\) is the historical average monthly levels at Jetty1. The variables \(n_{d,MH}, n_{d,ER}, n_{d,ON}, \) and \(n_{d,\text{Jetty1}}\), are normalizing constants equal to 1.3595e+011, 2.6011e+010, 5.5100e+010, and 2.695, respectively. Similar to the description for the upstream normalizing constants used in Component 1 above, the normalizing constants were calculated via a trial and error procedure. Again, note that the values of these normalizing constants are not critical since the optimization algorithms will adjust the two rule curve parameters to work well for these fixed values of the constants.

DownstreamStorage\_im of zero indicates that the water storage condition in the immediate downstream of a control point is normal; a positive storage value means the water available is greater than normal and a negative value indicates the available water is less than the normal condition. As can be seen in Figure 3, component 2 has two segments (with two different slopes) and the breakpoint is located at the storage of zero. The slope values, \(g\) and \(h\), are negative to reflect the inverse relationship between downstream storage and release. In other words, the less the downstream storage is the more the release is. The component 2 slope values \(g\) and \(h\) are parameters of the rule curve and should be determined through optimization.

**Component 3:**

Component 3 for control point \(i\) was designed to represent the storage condition downstream of the next control point (i.e., between control points \(i+1\) and \(i+2\)). The storage condition between control points \(i+1\) and \(i+2\) can be calculated through the following equations:

\[
\text{DownstreamStorage\_far}(1) = \frac{(ASC (Z_{SC} - \text{avg}Z_{SC}) + AER (Z_{ER} - \text{avg}Z_{ER}))}{n_{d,MH}} \tag{15}
\]

\[
\text{DownstreamStorage\_far}(2) = \frac{(AON (Z_{ON} - \text{avg}Z_{On}))}{n_{d,ER}} \tag{16}
\]

\[
\text{DownstreamStorage\_far}(3) = \frac{(Z_{\text{Jetty1}} - \text{avg}Z_{\text{Jetty1}})}{n_{d,ON}} \tag{17}
\]

Note that component 3 is not applicable for control point 4 being Lake Ontario regulation structure. The terms in above equations were already defined for equations 7-14.

Similar to Component 2, Component 3 has two negatively sloped segments with a breakpoint located at the water storage of zero (see Figure 3); however, unlike Component 2, Component 3 is not always active. Component 3 involves an if-then statement to be calculated as:

IF the lakes between control points \(i\) and \(i+1\) and the lakes between control points \(i+1\) and \(i+2\) have the same storage condition (both in shortage or both in excess)

\(\rightarrow\) calculate Component 3 from Component 3 equation (as shown in Figure 3)

ELSE Component 3 = 0
The logic behind this if-then statement is explained through an example. Suppose that at the beginning of a Superior release decision time interval Lake MH is in excess condition (wet) while the set of Lake St Clair and Lake Erie is in shortage condition (dry). In such case, Component 3 would become zero since any extra water released from Superior for Lake St. Clair and Lake Erie would first exacerbate the excess water condition on Lake MH. Instead, in such a case, the already available excess water on Lake MH can be released for downstream to Lake St Clair and Lake Erie.

Moreover, as defined in Figure 3, Component 3 has only one parameter, $a$, varying between 0 and 1. This parameter is multiplied by the slope parameters of Component 2, $g$ and $h$, to determine the slopes in Component 3 equation. As such, Component 3 is limited to have an equal or smaller impact (per unit change in storage condition) on release decision than Component 2.

2.4.2 Rule Curve Seasonality

Two regulation seasons in a year were considered for all control points in rule curve development. This means that we optimized two different sets of rule curves – one for each season. Rule curves therefore generate a target release that is a function of season as well as beginning of regulation time step levels and flows across the system. The regulation seasons are described as follows:

- For Lake Superior, the Summer Season is May through November, and Winter Season is December through April.
- For Lake MH and Lake Erie, ice-free season is 3rd quarter month of April through 3rd quarter month of December, and ice season is 4th QM of December to 2nd QM of April (suggested by David Fay of Environment Canada, personal communication)
- For Lake Ontario, shipping season is 4th quarter month of March through 3rd quarter month of December, non-shipping season is 4th quarter month of December through 3rd quarter month of March (suggested by D. Fay of Environment Canada, personal communication)
2.5 Optimization Formulation

As presented in Section 2.4.1, for a single season, the number of parameters to be optimized in the rule curve equations with four control points (Lake Superior, MH, Erie, and Ontario) are 10, 10, 10, and 9, respectively. As two seasons were assumed for regulation of each control point, there are 78 rule curve parameters (i.e., decision variables) in total to be optimized.

2.5.1 Primary Objective of Multi-lake Regulation

The ultimate objective of the project is to evenly enhance the status of water levels in the Great Lakes Saint Lawrence System so that all potential benefits resulting from any system improvement should be fairly shared among all and expenses associated with possible severe future climate should be fairly distributed. The water level at nine main components of the system including Lake Superior, Lake Michigan-Huron, Lake St. Clair, Lake Erie, Lake Ontario, Iroquois Headwater, Saunders Headwater, Pointe Claire on Lake St. Louis, and Jetty1 at Montreal were selected as performance evaluation points, each of which is a representative of a group of stakeholders.

The primary objective of the UW plan is to maintain the water level at these nine evaluation points within the range simulated for the historical period (1900-2008) when experiencing any future climate condition. A risk-based objective function was defined to keep water level in this desirable range, as going beyond the desirable range was deemed a failure. Table 1 reports the extreme levels (i.e., failure thresholds) over the historical period of 1900-2008 as simulated through the current system (i.e., with the current control structures) and current regulation plan (77A+58DD). As maintaining the water level in these specified ranges under a severe future climate is not always possible, the UW plan focuses to improve the system (i.e., reducing the frequency of being beyond extremes) over what is called “Base Case” in this report. Base Case provides a baseline of performance comparison and refers to performance of the system with only the current two structures (Lake Superior and Lake Ontario) that are operated/regulated under the current regulation plan. Overall, the objective function is constructed to try to identify a plan that improves upon the base case regulation performance (reduced frequency of going beyond extremes) at every evaluation location for every NBS scenario considered.

Initially, a single NBS scenario based objective function was developed and tested as a proof of concept. It was then extended to a multi-NBS scenario approach to cover different possible future severe conditions and circumstances. The following sub-sections present the details of both approaches.
Table 1. Min and Max monthly extreme simulated average levels at various evaluation locations for the historical period (1900-2008). Extremes based on simulation results provided by Environment Canada (personal communication, Yin Fan).

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>Location</th>
<th>Simulated Period</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lake Superior</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>181.70</td>
<td>183.80</td>
<td>182.80</td>
<td>183.80</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
<td>183.70</td>
</tr>
<tr>
<td>2</td>
<td>Michigan-Huron</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>182.91</td>
<td>182.81</td>
<td>182.80</td>
<td>182.80</td>
<td>182.80</td>
<td>182.80</td>
<td>182.80</td>
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<td>182.80</td>
<td>182.80</td>
<td>182.80</td>
</tr>
<tr>
<td>5</td>
<td>Lake St. Clair</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
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<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
</tr>
<tr>
<td>4</td>
<td>Lake Erie</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>174.08</td>
<td>174.08</td>
<td>174.08</td>
<td>174.08</td>
<td>174.08</td>
<td>174.08</td>
<td>174.08</td>
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<td>174.08</td>
</tr>
<tr>
<td>5</td>
<td>Lake Ontario</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
<td>175.78</td>
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<td>175.78</td>
<td>175.78</td>
</tr>
<tr>
<td>9</td>
<td>JETTY at Montev</td>
<td>1900-2008</td>
<td>Max Extreme</td>
<td>6.94</td>
<td>8.64</td>
<td>9.40</td>
<td>9.55</td>
<td>8.84</td>
<td>8.77</td>
<td>7.83</td>
<td>7.44</td>
<td>7.14</td>
<td>7.31</td>
<td>7.59</td>
<td>7.66</td>
</tr>
</tbody>
</table>

2.5.2 Single NBS Scenario Approach

A risk-based objective function was designed first to optimize the rule curve parameters for any single NBS scenario. As explained in Section 2.5.1, the system performance in the base case is the baseline in the objective function. Therefore, \( A \) is a given set of data for a specific NBS scenario as:

\[
A = \{ a_j | j = 1, 2, \ldots, n \} \tag{18}
\]

where \( a_j \) is the number of times in base case that the level at evaluation point \( j \) (obtained from plan 77A+58DD) is simulated beyond the range of the simulated historical extremes (in Table 1). Note that \( a_j \) corresponds to the risk of failure in the base case at evaluation point \( j \), \( Risk_{j_{BC}} \), such that

\[
Risk_{j_{BC}} = \frac{a_j}{T}
\]

where \( T \) is the total number of time steps in simulation. Failure in this context means levels are simulated outside of the historical period (1900-2008) simulated range.

Assuming \( X \) is vector containing the 78 decision variables (i.e., rule curve parameters), \( Y = f(X) \) is defined as a set of performance data, for a specific NBS scenario, and a specific new release plan \( (X) \). Like \( A \), the components of \( Y \) are:

\[
Y = \{ y_j | j = 1, 2, \ldots, n \} \tag{19}
\]

where \( y_j \) is the number of times under release plan \( X \) that the level at evaluation point \( j \) is simulated beyond the range of the simulated historical extremes (i.e., Lake Superior, Lake MH, \ldots). Similar to base case, \( y_j \) corresponds to the risk of failure of plan \( X \) at the evaluation point \( j \), \( Risk_{j}(X) \), such that

\[
Risk_{j}(X) = \frac{y_j}{T}
\]

where \( T \) is the total number of time steps in simulation.

Simply minimizing the average risk of any new plan will not achieve the desired objective to improve upon or maintain risk of failures relative to the base case for all evaluation points. For
example, an increased risk at one evaluation point relative to the base case is not generally acceptable to the project Study Board even if risk at one or more evaluation points is completely mitigated. Therefore, the following more complicated objective function is defined to achieve the desired objective:

\[
\text{Minimize } \sum_{j=1}^{n} z_j(y_j - a_j)^2 - \sum_{j=1}^{n} (1 - z_j)(a_j - y_j)^{0.5}
\]  

\[
Z_j = 0 \quad \text{if } y_j < a_j \quad \text{(performance in point } j \text{ better than baseline)}
\]

\[
Z_j = 1 \quad \text{if } y_j \geq a_j \quad \text{(performance in point } j \text{ worse than baseline)}
\]

Note that \(Z_j\) is computed by directly evaluating the “if” statements above for any given solution and thus, \(Z_j\) not a decision variable. The square and square root functions in the objective function help the optimizer balance the benefits and costs across all evaluation points in a way that:

- The first priority is given to the evaluation points which experience the highest level of inferiority compared to base case
- The second priority is given to the evaluation points which are still inferior compared to base case but the degree of inferiority is reduced
- The third priority which is completely dominated by first and second priorities is given to evaluation points which are slightly better than base case.
- The forth/last priority is given to the evaluation points which are substantially better than base case.

Note that the objective function value, as formulated above, does not have an immediately interpretable meaning. To fully interpret a given objective function value, it must be disaggregated into its components (i.e., risk values at each evaluation point). The only immediate interpretation is if this value is greater than zero for new release plan \(X\), there is at least one evaluation point at which state the risk of plan \(X\) is higher than the risk of base case. On the contrary, if the value is well below zero, then plan \(X\) outperforms the base case at every evaluation point.

### 2.5.3 Multi NBS Scenario Approach

The rule curve parameter values optimized through the single NBS scenario formulation may suffer from a lack of generalization. In other words, the rule curve would perform very satisfactorily on the NBS scenario used for optimization and other NBS scenarios with similar characteristics. However, it may perform poorly on NBS scenarios with significantly different behaviors. Thus, a multi NBS scenario formulation was developed to optimize the rule curve parameters over multiple NBS scenarios with significantly different behaviors simultaneously. The resulting rule curves would be expected to be robust and more reliable when facing unpredictable future climate conditions. The multi-scenario formulation is as follows:
\( n \): number of evaluation points (i.e., Lake Superior, Lake MH, \( \ldots \))

\( m \): number of NBS scenarios to optimize a single regulation plan for

\( A = \{a_{j,k} | j = 1, 2, \ldots, n \ \& \ k = 1, 2, \ldots, m\} \)

\( a_{j,k} \): base case performance on evaluation point \( j \) in scenario \( k \)

\( Y = \{y_{j,k} | j = 1, 2, \ldots, n \ \& \ k = 1, 2, \ldots, m\} \)

\( y_{j,k} \): UW plan performance on evaluation point \( j \) in scenario \( k \)

The multi-scenario objective function is:

\[
\text{Minimize } \sum_{j=1}^{n} \left\{ z_j \left( \max_k (y_{j,k} - a_{j,k}) \right)^2 - (1 - z_j) \left( \min_k (a_{j,k} - y_{j,k}) \right)^{0.5} \right\}
\]

(21)

\[ \text{if } \max_k (y_{j,k} - a_{j,k}) > 0 \text{ then } z_j = 1 \]

\[ \text{otherwise } \quad z_j = 0 \]

As can be seen, this optimization formulation is a \textit{minmax/maxmin} optimization problem trying to minimize the maximum inferiority and maximize the minimum improvement in a way that:

- If the new plan at evaluation point \( j \) performs worse than base case in at least one NBS scenario (there is at least one \( k \) such that \( y_{j,k} - a_{j,k} > 0 \)), the first term is activated in order to minimize the square of maximum inferiority.
- If the new plan at evaluation point \( j \) performs better than base case in all NBS scenario, maximize the square root of the minimum improvement.
- Like the single scenario objective function, the square function in the first term gives very high weights to high degradations to force them down as much as possible as a first priority.
- The square root function in the second term gives more weight to small improvements and less weight to large improvements. As such, increasing smaller improvements is of priority over larger improvements.
- Due to the inherent difference of square and square root functions (and as the improvements/degradations, \( y_{j,k} - a_{j,k} \), can only be integer), weights given to degradations are much higher that weights given to improvements making the optimization primarily focus on removing/improving the degradations.
- The square and square root functions in the objective function help the optimizer balance the benefits and costs across all evaluation points as described in Section 2.5.2.

A total of eight different NBS scenarios, selected from the 50,000 year LOSL NBS sequence described in Section 2.1, were identified by David Fay of Environment Canada. These scenarios were selected to be diverse in terms of generating a range of high and low lake levels overall, as well as differentially across, the Great Lakes when the base case regulation strategy was simulated. These eight NBS scenarios are described in more detail in Appendix 3. Seven of
these scenarios were the most extreme periods out of the 50,000 year NBS sequence with respect to different characteristics of simulated lake levels (one was randomly selected). Therefore, these eight scenarios were thought of as representative of possible extreme future NBS sequences under climate change.

2.5.4 Constraints in the Optimization Formulation

Beside the implicit constraints associated with the routing in the simulation model, it was necessary to consider bound constraints on the decision variables (i.e., rule curve parameters) as opposed to searching in an infinite space. Our selected optimization algorithm requires bounds on decision variable values and appropriately sized bound constraints (i.e., not overly conservative and thus too wide) can substantially enhance the computational efficiency of optimization. For the \( b \) and \( g \) parameter in component 1 and component 2, lower and upper bounds were selected by trial and error on the basis of average and standard deviation of associated factors (inputs) to the rule curves. Parameters \( a, c, \) and \( d \) for component 1 were assumed to be proportional to \( b \) and vary in the ranges of \([2b – 10b], [0.5b - 1.5b], \) and \([2c – 10c], \) respectively. The range for parameter \( f \) in component 1 was defined such that \( f \) would approximately fall between a storage condition of 0 and the maximum observed storage excess. The range for the parameter \( e \) was handled in a parallel fashion. Parameter \( h \) was also assumed proportional to \( g \) and to vary in \([0.5g – 1.5g]\). Moreover, as stated earlier, the \( \alpha \) parameter in component 3 can vary in the range of zero and one. Appendix 2 in list all 78 decision variables/rule curve parameters that are optimized along with their associated ranges.

As mentioned in 2.4.1, the release values calculated from equation (1) are unitless numbers with an unbounded range. First, the unitless target releases are bounded to a 0-1 range such that any unitless target release calculated over 1 is set to a value of 1. Then, this normalized 0-1 target release should be transformed into a target release in units of m\(^3\)/s. It should be noted that we rely on the optimization algorithm to identify rule curve parameters that allow our convenient scaling procedure to work well. The allowable target release range from Lake Superior is set at 280 m\(^3\)/s to 4010 m\(^3\)/s (corresponding to normalized target release values of 0 and 1, respectively) for the summer season and 1150 m\(^3\)/s to 2410 m\(^3\)/s for winter based on physical limits for the existing structures on the St. Marys River. The release range for Lake MH and Lake Erie is assumed to be 200 m\(^3\)/s to 12000 m\(^3\)/s for both seasons. These ranges allow for the biggest possible impact of regulation (least constrained problem). The allowable release range for Lake Ontario is 4810 m\(^3\)/s to 10010 m\(^3\)/s for the shipping season and 4360 m\(^3\)/s to 9490 m\(^3\)/s for non-shipping season based on the physical and operational constraints of existing structures along the Upper St. Lawrence River. As noted in Section 2.2, the target release values in these ranges are subject to change (resulting in an actual release) to respect the physical/hydraulic constraints of the controlling structures.

In addition to the range constraints on Lake Superior outflows, the Study Board for the Lake Superior Plan Formulation part of this IUGLS study indicated critical lake levels for Lake Superior (monthly average minimum and maximum of 182.76 m and 183.86 m, respectively)
that all proposed Superior regulation plans should try to respect. For this multi-lake regulation work, we also tried to respect these critical lake levels by handling it as a constraint of secondary importance. The Superior release constraints described above were always respected. If possible, the following logic was applied to modify the target release in units of m$^3$/s as follows. Note that all variables below can be defined based on equivalent depths:

\[
\begin{align*}
\text{IF } Z_{\text{Sup}}(t) + \text{Max}_N\text{BS}_{\text{Sup}}(t) - \text{Target Release}(t) &> 183.86 \text{ THEN} \\
\quad \text{New Target Release}(t) = Z_{\text{Sup}}(t) + \text{Max}_N\text{BS}_{\text{Sup}}(t) - \text{Max}_Z_{\text{Sup}} \\
\text{IF } Z_{\text{Sup}}(t) + \text{Min}_N\text{BS}_{\text{Sup}}(t) - \text{Target Release}(t) &< 183.00 \text{ THEN} \\
\quad \text{New Target Release}(t) = Z_{\text{Sup}}(t) + \text{Min}_N\text{BS}_{\text{Sup}}(t) - \text{Min}_Z_{\text{Sup}}
\end{align*}
\]

The first if statement says that if the maximum historical NBS for month $t$, $\text{Max}_N\text{BS}_{\text{Sup}}(t)$, was added to the current month’s beginning lake level, $Z_{\text{Sup}}(t)$, and the planned target release, $\text{Target Release}(t)$, was not enough to avoid exceeding 183.86 m, then the a new target release should be computed to try to ensure 183.86 is not exceeded.

Similarly, the second if statement says that if the minimum historical NBS for month $t$, $\text{Min}_N\text{BS}_{\text{Sup}}(t)$, was added to the current month’s beginning lake level, $Z_{\text{Sup}}(t)$, and the planned target release, $\text{Target Release}(t)$, was too little to avoid falling below 183.00 m, then the a new target release should be computed to try to ensure the lake level does not fall below 183.00. Preliminary testing showed that 183.00 was necessary to trigger a new decision instead of the soft lower lake level constraint of 182.76 m.

2.5.5 Regulation Cost as a Another Objective

Controlling the outflow of Lakes MH and Erie is costly. If the controlled outflow of the lake is higher than the natural connecting channel flow at the same upstream and downstream lake levels then excavation is required to increase the conveyance capacity of the connecting channel. If the controlled outflow is less than the natural connecting channel flow then a structure is required to hold back the water on the lake.

Structural and excavation costs were estimated from cost estimates in the Levels Reference Study, Annex 3 (Working Committee 3, LRS, 1993) pg. 6-50 and updated to 2010 dollars in Appendix B of Bruxer and Carlson (2010). It is assumed that a control structure on St. Clair River to hold back the water on Lake MH costs $513.1 million and a control structure on Niagara River to hold back the water on Lake Erie costs $533.2 million. These structural costs are assumed constant for all degrees of flow reduction relative to natural connecting channel flow and for any range of lake levels. Table 2 and Table 3 report the excavation costs as a function of increased flow in the St. Clair River and Niagara Rivers, respectively.
Table 2. Excavation Cost for Increasing Conveyance of St. Clair River where the Increase in Q is the maximum increase in flow over natural channel flow at the same conditions.

<table>
<thead>
<tr>
<th>Increase in Q (cms)</th>
<th>Cost (billions 2010 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>142</td>
<td>0.44</td>
</tr>
<tr>
<td>283</td>
<td>0.88</td>
</tr>
<tr>
<td>425</td>
<td>1.32</td>
</tr>
<tr>
<td>566</td>
<td>1.76</td>
</tr>
<tr>
<td>850</td>
<td>2.42</td>
</tr>
<tr>
<td>1416</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Table 3. Excavation Cost for Increasing Conveyance of Niagara River where the Increase in Q is the maximum increase in flow over natural channel flow at the same conditions.

<table>
<thead>
<tr>
<th>Increase in Q (cms)</th>
<th>Cost (millions 2010 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>566</td>
<td>88</td>
</tr>
<tr>
<td>656</td>
<td>352</td>
</tr>
<tr>
<td>736</td>
<td>528</td>
</tr>
<tr>
<td>850</td>
<td>616</td>
</tr>
<tr>
<td>991</td>
<td>704</td>
</tr>
<tr>
<td>1303</td>
<td>792</td>
</tr>
<tr>
<td>1642</td>
<td>880</td>
</tr>
<tr>
<td>2265</td>
<td>968</td>
</tr>
<tr>
<td></td>
<td>1012</td>
</tr>
</tbody>
</table>

The excavation cost can be estimated by interpolating (or extrapolation if required) the largest “Increase in Q” of the St. Clair River from Table 2 and from Table 3 for the Niagara River. Summation of these two costs in addition to the cost of control structures yields an estimate of the total cost of regulation associated with controlling the outflow of Lakes MH and Erie. Equation (22) below provides the total cost estimation.

Cost Objective

\[
\text{Cost} = f_{\text{exc}}^{\text{SCR}} \left( \max_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right) \right)_{\text{SCR}} + f_{\text{exc}}^{\text{SCR}} \left( \max_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right) \right)_{\text{NiR}} + f_{\text{str}}^{\text{SCR}} \left( \min_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right) \right)_{\text{SCR}} + f_{\text{str}}^{\text{SCR}} \left( \min_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right) \right)_{\text{NiR}}
\]

(22)

where \( f \) is the cost function, superscripts \( \text{exc} \) and \( \text{str} \) distinguish between excavation cost and cost of building a structure to hold back the water, respectively, and subscripts \( \text{SCR} \) and \( \text{NiR} \) denote St. Clair and Niagara Rivers respectively. Here, \( f_{\text{exc}}^{\text{SCR}} \) is calculated by piecewise linear interpolation (extrapolation if required) of the maximum positive difference between connecting channel flow generated by the rule curve, \( Q_{\text{rule}}^t \), and natural flow in the connecting channel \( Q_{\text{natural}}^t \), in each simulation time step \( t \) (one day in our analysis) from Table 2 or Table 3. \( Q_{\text{natural}}^t \)
is calculated by the open-water lake-to-lake stage-fall-discharge equations that describe the current conveyance of the St. Clair and Niagara Rivers in CGLRRM (see Appendix 4 for equations). Also, $f_{str}$ is a step function for both SCR and NiR that assigns a constant structural cost ($513.1$ million for SCR and $533.2$ million for NiR) to any reduction in rule curve based flow relative to the natural flow (negative flow difference in the last two terms of the equation).

The cost estimates above are based on a specific set of alternative structures and a limited range of connecting channel flow changes. The magnitude of flow changes considered in this report are not constrained, and as results will show, are well beyond the range of conditions considered when these costs were estimated in the Levels Reference Study (Levels Reference Study Board, 1993). Furthermore, the cost estimates for excavation of the Niagara River were noted in Annex 3 of the Levels Reference Study (Working Committee 3, LRS, 1993) to be likely inaccurate for the highest flow changes. Given these factors, it seems clear that any cost estimates based on the above sources are rough and are almost certainly underestimated if extrapolation beyond the data points in Table 2 and Table 3 is required.

### 2.6 Optimization Algorithms

The various single objective optimization problems solved in this study based on the formulation presented above (Equations 20 or 21) involve optimizing up to 78 rule curve parameters and the evaluation of each solution requires a computationally intensive model simulating the entire Great Lakes – St. Lawrence system over multiple NBS scenarios. Each solution evaluation via this model (i.e., system simulation over the eight NBS scenarios) takes about 2.7 minutes to execute on a 2.8GHz Intel Pentium processor with 2 GB of RAM and running the Windows XP operating system. Thus, a highly efficient optimization algorithm, Dynamically Dimensioned Search (DDS) developed by Tolson and Shoemaker (2007), which has been designed for optimization problems with many decision variables and a limited number of objective function (solution) evaluations, was used to solve the optimization problem. Through preliminary testing, it was recognized that DDS requires at least 3,000 function evaluations (solution evaluations) in this experiment to converge to a desirable region of attraction; therefore, an optimization trial with DDS takes about 5.5 days (~3,000 * 2.7 minutes) of serial computation. To further improve the computational efficiency, the model pre-emption strategy developed by Razavi et al. (2010) was also used in conjunction with the DDS algorithm. The pre-emption strategy was able to reduce the computational time by almost 50 percent.

As the optimization problem formulated here is very complex and multi-modal, all single objective optimization problems were solved using four to six independent optimization trials conducted with different random seeds to try to ensure that the final best solution out of these six would be reasonable good quality solution (or at least not a single unlucky or poor result from the algorithm). The best solution of these trials was typically further optimized by subsequent DDS optimization trials initialized to the best solution.
Multi-objective optimization problems are also formulated to approximate the tradeoff between the frequency-based (or risk-based) objective (Equation 21) and the new structures total cost (Equation 22). These bi-objective optimization problems also involve optimizing 78 rule curve parameters and the evaluation of each solution requires a computationally intensive model simulating the entire Great Lakes – St. Lawrence system over multiple NBS scenarios. Each solution evaluation via this model (i.e. system simulation over the eight NBS scenarios) takes about 2.7 minutes to execute on a 2.8GHz Intel Pentium processor with 2 GB of RAM and running the Windows XP operating system. These problems are solved by the Pareto Archived Dynamically Dimensioned Search (PA-DDS) optimization algorithm developed by Asadzadeh and Tolson (2009). PA-DDS is a heuristic multi-objective optimization algorithm based on the single objective DDS algorithm introduced by Tolson and Shoemaker (2007) for computationally intensive hydrologic model calibration problems. PA-DDS conducts the search in the exact same way as DDS does; moreover, it archives all the non-dominated (tradeoff) solutions during the search. Also, whenever PA-DDS cannot find a new non-dominated solution, it uses the crowding distance metric (Deb et al. 2002) to select a current non-dominated solution as the center of the search neighbourhood. PA-DDS inherits the parsimonious characteristic of DDS, so it has only one algorithm parameter and all PA-DDS runs apply the same (default) algorithm parameter value (r=0.2). Multiple PA-DDS optimization trials are conducted and the results are aggregated to show the non-dominated or tradeoff solutions.
3 Results

The first results presented in Section 3.1 show that the current base case system management of Plan 77A & Plan 55DD does not perform well under various NBS scenarios that may be representative of future extreme climates. Section 3.2.1 then details how optimizing releases with the current suite of structures will not improve system performance appreciably while Section 3.2.2 details the substantial system performance improvements possible with two new control structures (St. Clair River and Niagara River). Sections 3.2.4 and 3.2.5 also assess the impact of including the lower St. Lawrence River evaluation points in the formulation. Multi-objective optimization models are solved in Section 3.3 to identify a more realistic (cost effective) solution and results show very good improvements with substantial cost savings over the best solution that ignores costs of new structures and the associated dredging. Section 3.4 summarizes findings with respect to managing Montreal water levels and Section 3.5 fully analyzes the performance of an example multi-lake regulation plan under the entire 50,000 year stochastic LOSL NBS data set.

3.1 Simulation of base case system management for future NBS scenarios

The base case management plan was simulated for the entire 50,000 year stochastic LOSL NBS data set with initial lake levels set to Jan 1, 1900 levels. The statistics on plan performance summarized in the following tables and figures. Table 4 reports the frequency of violating extremes (upper or lower) as well as the frequencies associated to lower extreme and upper extreme separately. Lake MH and Lake St. Clair are the most critical evaluation points with failure frequencies at 2.59% and 2.75%, respectively. Further analysis of this simulation run shows that 6.41% of the time, water levels for at least one evaluation point will be beyond the historical period simulated extremes.

The longest periods when the surface elevation at each evaluation point is continuously lower than the historical minimum extreme and continuously higher than the historical maximum are calculated for the base case management plan for the 50,000 year simulation in Table 5. These period lengths are one way to describe system resilience (how long the system remains in a failure state). Table 5 shows there are some extremely long periods of continuously extreme lake levels. For example, Lake MH has one period of 169 consecutive months (14 years) where water levels are lower than the historical period extremes.
Table 4- Frequency of violating the monthly simulated extremes for the historical period (1900-2008) based on simulation of full 50,000 year stochastic LOSL NBS data set under base case management.

<table>
<thead>
<tr>
<th>Evaluation point</th>
<th>Base Case</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency of Violating Extremes</td>
<td>Frequency of Violating Upper Extreme</td>
<td>Frequency of Violating Lower Extreme</td>
<td></td>
</tr>
<tr>
<td>Lake Superior</td>
<td>2.31%</td>
<td>1.60%</td>
<td>0.71%</td>
<td></td>
</tr>
<tr>
<td>Lake MH</td>
<td>2.59%</td>
<td>0.52%</td>
<td>2.07%</td>
<td></td>
</tr>
<tr>
<td>Lake St Clair</td>
<td>2.75%</td>
<td>0.42%</td>
<td>2.33%</td>
<td></td>
</tr>
<tr>
<td>Lake Erie</td>
<td>1.98%</td>
<td>0.56%</td>
<td>1.42%</td>
<td></td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>0.87%</td>
<td>0.31%</td>
<td>0.56%</td>
<td></td>
</tr>
<tr>
<td>IHW</td>
<td>0.64%</td>
<td>0.40%</td>
<td>0.24%</td>
<td></td>
</tr>
<tr>
<td>SHW</td>
<td>0.03%</td>
<td>0.00%</td>
<td>0.03%</td>
<td></td>
</tr>
<tr>
<td>Pte Claire</td>
<td>0.79%</td>
<td>0.16%</td>
<td>0.63%</td>
<td></td>
</tr>
<tr>
<td>Jetty 1</td>
<td>0.30%</td>
<td>0.09%</td>
<td>0.20%</td>
<td></td>
</tr>
<tr>
<td>All 9 above evaluation points</td>
<td>6.41%</td>
<td>2.68%</td>
<td>3.74%</td>
<td></td>
</tr>
</tbody>
</table>

1. Consider violation of extremes to be a violation at one or more evaluation points (e.g. at least one evaluation point from Superior through Jetty 1 is violating the 1900-2008 period extremes).

Table 5. Longest periods in 50,000 years when water surface elevation at each evaluation point is lower than the minimum extreme and higher than the maximum extreme.

<table>
<thead>
<tr>
<th>Base Case</th>
<th>Extreme</th>
<th>SP</th>
<th>MH</th>
<th>SC</th>
<th>ER</th>
<th>ON</th>
<th>IHW</th>
<th>SHW</th>
<th>PCL</th>
<th>JET</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min</td>
<td>45</td>
<td>169</td>
<td>98</td>
<td>97</td>
<td>65</td>
<td>23</td>
<td>5</td>
<td>20</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>109</td>
<td>63</td>
<td>20</td>
<td>21</td>
<td>19</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>6</td>
</tr>
</tbody>
</table>

In order to assess system performance on extreme NBS scenarios the base case management plan was simulated for the eight scenarios described in Appendix 3 with initial lake levels set to Jan 1, 1900 levels. Figure 4 shows that high frequencies of extreme lake levels are possible under a number of extreme NBS scenarios. For example, Lake MH and Lake St. Clair are simulated to experience frequencies of new extreme levels for between 20%-30% of all months simulated in NBS scenarios 1, 3 and 7.

Overall, results above show that new extreme lake levels will be experienced at unacceptably high frequencies and for long periods of time (particularly for NBS extreme scenarios). As such, the following sections evaluate what kind of improvement in system performance is possible by changing the release policies for the existing structures (St. Mary’s and St. Lawrence) as well as adding new structures (St. Clair River and Niagara River) to the system.
Figure 4- Performance of base case management plan over the eight NBS scenarios. IHW and SHW evaluation points are combined as one point as are Pointe Claire and Jetty1.
3.2 Single-objective optimization for frequency-based objective

3.2.1 Two-point control formulation

A new management plan with two control points at the existing structure locations (St. Mary’s River and St. Lawrence River), called the 2-point UW plan, was optimized using the objective function (Equation 21) and constraints described in Sections 2.5.4. The 2-point UW plan was optimized with seven evaluation points (Lake Superior, Lake MH, Lake St. Clair, Lake Erie, Lake Ontario, Iroquois + Saunders headwaters) in the objective function. Note that the Iroquois and Saunders locations are combined to define one evaluation point with the same weight or importance as each upstream lake. Pointe Claire and Jetty1 locations are left out of the formulation initially given that their inclusion will only make the problem more difficult and the expectation was that the current suite of two control points would not by themselves be capable of achieving the objective to improve performance everywhere, in all NBS scenarios.

The rule curves for releases from Lake Superior and Lake Ontario have a very similar structure as the multi-lake rule curves in Section 2.4.1. One difference is that for the Lake Ontario rule curve, component 1 upstream storage condition (see Figure 2) does not include the upstream inflow to Lake Ontario (since it is unknown when Lake Erie outflow is not controlled). The only other difference is that for Lake Superior releases, the component 2 downstream storage condition (see Figure 3) is based on Lake Michigan-Huron through to Lake Erie.

Figure 5 shows the comparative results of the 2-point UW plan, excluding Montreal (Lower St. Lawrence River at Montreal evaluation points not considered), optimized through the multi-NBS scenario optimization framework over the eight NBS scenarios. The performance of the 2-point UW plan versus the base case plan for each NBS scenario is shown in eight individual subplots in Figure 5. For example, the first subplot in Figure 5 shows the frequency of going beyond the historical simulated extremes at the six evaluation points (two points aggregated to one point such that the six data points in the figure have the same weight) considered in the objective function for each plan (two series). Note that the frequency of going beyond the historical simulated extremes is also called the frequency of violation in this report and this frequency is the ratio of the number of months when the average level goes beyond the monthly historical simulated extremes over the total number of months in that scenario. As can be seen in Figure 5, the 2-point UW plan is unable to perform noticeably better than the base case plan and the UW plan certainly is unable to improve performance at all evaluation points in all scenarios.

Table 6 also reports the improvement of UW plan over base case (in percent) at different evaluation points under different NBS scenarios. The percent improvement was calculated through the following equation:

\[
\text{Improvement(\%)} = \frac{\text{base case violation frequency} - \text{UW plan violation frequency}}{\text{base case violation frequency}} \times 100
\]  

(23)
Figure 5- Performance of 2-point UW plan (Lower St. Lawrence River at Montreal evaluation points not considered in objective function) over the eight NBS scenarios in comparison with base case.
The results in Table 6 demonstrate that relative to the base case management of the system, changing the way release decisions are made at the existing structures will not improve system performance (keeping levels within their historical period simulated extreme ranges as much as possible). For example, when averaging across all evaluation points and all scenarios, the average %Improvement is -11%. Note that the %Improvement measure in Table 6 is somewhat misleading in some cases. For example, the average improvement of the 2-point UW plan would be 12% if the two inflated numbers for IHW+SHW were ignored. Since including additional evaluation points in the objective function will only make this problem more difficult to solve, the formulation with Pointe Claire and Jetty1 evaluation points is not considered for the 2-point UW plan.

Table 6- Improvement\(^1\) of the 2-point UW plan (Lower St. Lawrence River at Montreal evaluation points not considered in objective function) at each evaluation point for each NBS scenario relative to base case

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>33%</td>
<td>22%</td>
<td>11%</td>
<td>18%</td>
<td>-33%</td>
<td>-9%</td>
<td>-14%</td>
<td>54%</td>
<td>10%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>1%</td>
<td>-10%</td>
<td>-2%</td>
<td>-5%</td>
<td>0%</td>
<td>-8%</td>
<td>2%</td>
<td>1%</td>
<td>-3%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>2%</td>
<td>-9%</td>
<td>-1%</td>
<td>-6%</td>
<td>0%</td>
<td>2%</td>
<td>3%</td>
<td>-1%</td>
<td>-1%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>3%</td>
<td>0%</td>
<td>3%</td>
<td>-13%</td>
<td>6%</td>
<td>-2%</td>
<td>-4%</td>
<td>-11%</td>
<td>-3%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>47%</td>
<td>0%</td>
<td>31%</td>
<td>6%</td>
<td>32%</td>
<td>-25%</td>
<td>45%</td>
<td>10%</td>
<td>18%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>43%</td>
<td>75%</td>
<td>40%</td>
<td>-600(^2)</td>
<td>42%</td>
<td>-480(^3)</td>
<td>88%</td>
<td>80%</td>
<td>-89%</td>
</tr>
<tr>
<td>Average</td>
<td>21%</td>
<td>13%</td>
<td>14%</td>
<td>-100%</td>
<td>8%</td>
<td>-87%</td>
<td>20%</td>
<td>22%</td>
<td>-11%</td>
</tr>
</tbody>
</table>

1. \(\%\text{ Improvement} = 100[\text{Base Case}_{\text{extreme violation count}} - \text{UW}_{\text{extreme violation count}}]/\text{Base Case}_{\text{extreme violation count}}\)
2. Inflated due to very small number of extreme violation counts (18 for UW plan versus 2 in base case).
3. Inflated due to very small number of extreme violation counts (29 for UW plan versus 5 in base case).

3.2.2 Four-point control formulation without Montreal

Given the inability to improve system performance with the existing structures in Section 3.2.1 above, two additional hypothetical structures on the St. Clair River and Niagara Rivers were considered. A UW plan with four control points was optimized using Equation 21 as the objective function (see Section 2.5.3) and subject to the constraints described in Sections 2.5.4. The plan was optimized with only six evaluation points (Lake Superior, Lake MH, Lake St. Clair, Lake Erie, Lake Ontario, and Iroquois + Saunders headwaters) in the objective function. This plan is commonly referred to as the 4-point UW plan (without Montreal). Note that the Iroquois and Saunders locations on the upper St. Lawrence are combined to define one evaluation point with the same weight or importance as each upstream lake. In this first version, the Lower St. Lawrence River evaluation points at Montreal (Pointe Claire and Jetty1) are left out of the objective to calculate the maximum possible benefit of the UW plan. With this approach, we roughly assume that structures could be built downstream of Montreal and operated to completely mitigate water level extremes on the Lower St. Lawrence. Explicitly
including additional structures on the St. Lawrence River is beyond the scope of this project. Section 3.2.3 repeats this optimization with Montreal as an evaluation point included in the objective function.

Figure 6 shows the comparative results of the 4-point UW plan (without Montreal) optimized through the multi-NBS scenario optimization framework. The performance of the 4-point UW plan versus the base case plan for each NBS scenario is shown in eight individual subplots in Figure 6. For example, the first subplot in Figure 6 shows the frequency of going beyond the historical simulated extremes at the six evaluation points considered in the objective function for each plan (two series). As can be seen in Figure 6, the 4-point UW plan substantially outperforms the base case at all evaluation points and in all scenarios. Table 7 also reports the improvement of the 4-point UW plan over base case (in percent) at different evaluation points under different NBS scenarios.

Overall observations from Figure 6 and Table 7 can be summarized as follows:

- Overall improvement (averaged) at all evaluation points and in all NBS scenarios is 68% for this 4-point UW plan.
- Improvement at Lake Erie is 100% suggesting that the 4-point UW plan can always keep Lake Erie level within the range of extremes simulated for the historical period.
- Improvement at Lake Ontario is the smallest of all evaluation points (35% averaged over all NBS scenarios).
- Strictly speaking, this 4-point UW plan does not meet the objective of improving performance at all evaluation points for all NBS scenarios (%Improvement is -17% for Lake Ontario in NBS Scenario 2). However, in comparison with the base case, this plan generates only one additional month (out of 840) where Lake Ontario goes beyond it’s historical simulated extremes. Therefore, for all intents and purposes, this 4-point UW plan does meet the objective of improving/maintaining performance at all evaluation points for all NBS scenarios. In fact, validation performance assessed in the next section demonstrates improved performance on Lake Ontario.
Figure 6- Performance of 4-point UW plan (Lower St. Lawrence River at Montreal evaluation points not considered in objective function) over the eight NBS scenarios in comparison with base case.
Table 7- Improvement\(^1\) of 4-point UW plan (Lower St. Lawrence River at Montreal evaluation points not considered in objective function) at each evaluation point for each NBS scenario relative to base case

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>89%</td>
<td>82%</td>
<td>79%</td>
<td>80%</td>
<td>87%</td>
<td>70%</td>
<td>100%</td>
<td>100%</td>
<td>86%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>50%</td>
<td>78%</td>
<td>58%</td>
<td>93%</td>
<td>43%</td>
<td>56%</td>
<td>49%</td>
<td>66%</td>
<td>62%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>73%</td>
<td>30%</td>
<td>80%</td>
<td>23%</td>
<td>72%</td>
<td>19%</td>
<td>90%</td>
<td>69%</td>
<td>57%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>62%</td>
<td>-17(^2)%</td>
<td>55%</td>
<td>22%</td>
<td>22%</td>
<td>24%</td>
<td>80%</td>
<td>30%</td>
<td>35%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>86%</td>
<td>50%</td>
<td>49%</td>
<td>50%</td>
<td>60%</td>
<td>40%</td>
<td>100%</td>
<td>100%</td>
<td>67%</td>
</tr>
<tr>
<td>Avg.</td>
<td>77%</td>
<td>54%</td>
<td>70%</td>
<td>62%</td>
<td>64%</td>
<td>52%</td>
<td>87%</td>
<td>77%</td>
<td>68%</td>
</tr>
</tbody>
</table>

1. % Improvement = \(100\left[\text{Base Case extreme violation count - UW extreme violation count}\right]/\text{Base Case extreme violation count}\)
2. The number of violations in base case is 6 (frequency=0.71%) while the same number in UW plan is 7 (frequency= 0.83%)

3.2.3 Validation performance of 4-point UW plan (without Montreal) and Base Case over the full 50,000 year LOSL NBS sequence

The base case and 4-point UW plan (without Montreal evaluation points in the objective function) were simulated for the entire 50,000 year stochastic NBS sample with initial lake levels set to the year 1900 levels. Statistics on plan performance with regards to the frequency of violating monthly extreme lake levels, the magnitudes of these violations, the distribution of lake levels and the distribution connecting channel flows were analyzed. These are summarized in the following Tables and Figures.

Table 8 reports the frequency of violating extremes (upper and lower, as defined in Table 1) as well as the frequencies associated to lower extreme and upper extreme violations separately. The total violation frequencies are substantially lower for the 4-point UW plan (without Montreal) compared to the base case except for Lake St. Clair. Lake St. Clair outflows are not controlled and as such Lake St. Clair levels can fluctuate greatly as a result of the controlled Lake MH outflows. These higher frequencies associated to the 4-point UW plan largely violate the upper extreme lake levels (frequencies are lower than base case for violating lower extremes) and this is expected as Lake St. Clair is not sufficiently large to dampen the very large flows now possible from Lake MH that are generated by the 4-point UW plan. Overall, for the base case, 6.2% of months are simulated such that at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes. This number is reduced to 3.8% for the 4-point UW plan and the majority of those violations are due to Lake St. Clair as can be seen in Table 8. In fact, if Lake St. Clair is ignored, the 4-point UW plan simulates 1.0% of months such that at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes compared to 5.5% of months in the base case.
Table 8- Frequency of violating the monthly simulated extremes for the historical period (1900-2008) based on simulation of full 50,000 year stochastic LOSL NBS data set for select plans.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>4-point UW Plan (without Montreal) from Section 3.2.2</th>
<th>Base Case</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency of Violating Extremes</td>
<td>Frequency of Violating Upper Extreme</td>
</tr>
<tr>
<td>Lake Superior</td>
<td>0.16%</td>
<td>0.12%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>0.67%</td>
<td>0.08%</td>
</tr>
<tr>
<td>Lake St Clair</td>
<td>2.99%</td>
<td>2.33%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>0.35%</td>
<td>0.16%</td>
</tr>
<tr>
<td>IHW</td>
<td>0.03%</td>
<td>0.01%</td>
</tr>
<tr>
<td>SHW</td>
<td>0.09%</td>
<td>0.00%</td>
</tr>
<tr>
<td>All 7 above evaluation points'</td>
<td>3.8%</td>
<td>-</td>
</tr>
<tr>
<td>All 7 evaluation pts except Lake St Clair</td>
<td>1.0%</td>
<td>-</td>
</tr>
</tbody>
</table>

1. Consider violation of extremes to be a violation at one or more evaluation points (e.g. at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes).

Figure 7 and Figure 8 show the histograms of the magnitude of extreme violations at all evaluation points. As an example, Lake Superior when operated with the 4-point UW plan (without Montreal) goes beyond the upper extremes (see upper left subplot in Figure 7) by 0-0.05 metres in 397 of the 600,000 (50,000 * 12) months in the simulation, while in the base case it goes beyond the upper extreme by 0-0.05 metres in 3567 months. For SHW, since we did not assume an upper extreme as discussed in Section 2.5.1, this figure only has the histogram plot for SHW when the water level violates lower extreme. Figure 7 and Figure 8 show that the 4-point UW plan (without Montreal) generates substantially lower numbers in each bin for all evaluation points, except for Lake St. Clair when violating the upper extreme and Saunders Headwater lower extreme. For example, the 4-point UW plan generates no extreme violations for Lake Erie and very few violations on Lake Superior and IHW. Overall, these histograms show that in addition to reducing the frequency of extreme violations, the magnitudes of the extreme violations are also substantially reduced by the 4-point UW plan (despite the fact that these magnitudes are not considered in the optimization formulation).
Figure 7. Histograms of monthly average extreme violation magnitudes for Lake Superior through Lake Erie evaluation points for the 50,000 year simulation under the 4-point UW (without Montreal) and base case plans – total number of time intervals (months) is 600,000.
Figure 8. Histograms of monthly average extreme violation magnitudes for Lake Ontario through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the 4-point UW (without Montreal) and base case plans – total number of time intervals (months) is 600,000.

Figure 9 compares the 4-point UW (without Montreal) plan to the base case in terms of the percentiles of water levels at all evaluation points (Lake Superior through Saunders headwaters) while Figure 10 compares the UW plan to the base case in terms of the percentiles of connecting channel flows (St. Marys River through Lake Ontario outflow). In Figure 9, the 4-point UW plan clearly compresses the range of water levels simulated over the 50,000 year period for all evaluation points except for Lake St. Clair and Saunders headwater. In these two exceptions, the range of lake levels are approximately the same in the 4-point UW plan and the base case and both Lake St. Clair high and low levels are higher relative to base case. Figure 10 shows that the
compressed water level ranges in the 4-point UW plan appear to require expanding the range of flows experienced in the connecting channels (except for Lake Ontario outflow).

Figure 9. Select quantiles from the frequency distribution of monthly average water levels for Lake Superior through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the 4-point UW (without Montreal) and base case plans.
Figure 10. Select quantiles from the frequency distribution of monthly average connecting channel flows for the St. Marys River through Lake Ontario Outflow for the 50,000 year simulation under the 4-point UW (without Montreal) and base case plans. Note that there is no structure controlling Lake St. Clair (Detroit River) outflows in the UW plan.

3.2.4 Four-point control formulation including Montreal

Another 4-point control plan was optimized that added another evaluation point to the objective function for the Lower St. Lawrence River at Montreal (Pointe Claire and Jetty1). The optimization problem solved here was exactly the same as the one solved in Section 3.2.2 except for the inclusion of the lower St. Lawrence River locations in the objective function. Note that the Pointe Claire and Jetty1 locations are combined to define one evaluation point with the same weight or importance as each upstream lake. This plan is commonly referred to as the 4-point UW plan (with Montreal). In comparison with the previous 4-point UW plan (without Montreal) in Section 3.2.2, this analysis determines the extent to which extreme levels at Montreal might be mitigated by control structures only upstream of Montreal and highlights the associated impacts.
on performance at upstream evaluation points. In other words, it is assumed that no control structures would be added to the lower St. Lawrence to control water levels at Montreal.

Figure 11 compares the 4-point UW plan (with Montreal) in the objective function and the 4-point UW plan (without Montreal) from Section 3.2.2. Note the new evaluation point at Montreal (Pointe Claire and Jetty1) is now in each of the subplots in Figure 11. Table 9 reports the improvement of the 4-point UW plan (with Montreal) over base case (in percent) at different evaluation points under different NBS scenarios and can be compared directly with Table 7. Observations from Figure 11 and Table 9 can be summarized as follows:

- Not surprisingly, when Montreal is not included in the objective function, Figure 11 shows that performance at Montreal is severely compromised. In contrast, the 4-point UW plan (with Montreal) generates better performance. However, even the 4-point UW plan (with Montreal) generates performance at Montreal that is often degraded relative to the base case (Scenarios 2, 4, 6, 8). It is interesting to note that Montreal performance is approximately equal or better than the base case in all of the low NBS scenarios (1, 3, 5, 7).
- Performance at Montreal is approximately the same or slightly worse than the base case across the eight NBS scenarios when it is included in the objective function.
- Managing for Montreal using only the upstream four control points causes some degradation in UW plan performance at upstream evaluation points (average of 34% improvement compared to 68% in Table 7 for upstream evaluation points).
- Most degradation of upstream performance after considering Montreal in the objective is observed at Lake MH and then at Lake Superior. This observation might be solution specific, and there may be other solutions to the same optimization problem being less affected by considering Montreal.
Figure 11- Performance of the 4-point UW plan (with Montreal) and 4-point UW plan (without Montreal) for the eight NBS scenarios in comparison with base case.
Table 9- Improvement\(^1\) of the 4-point UW plan (with Montreal) at each evaluation point for each NBS scenario relative to base case.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>83%</td>
<td>63%</td>
<td>66%</td>
<td>58%</td>
<td>73%</td>
<td>4%</td>
<td>29%</td>
<td>92%</td>
<td>58%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>40%</td>
<td>65%</td>
<td>12%</td>
<td>27%</td>
<td>26%</td>
<td>8%</td>
<td>3%</td>
<td>42%</td>
<td>28%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>75%</td>
<td>64%</td>
<td>79%</td>
<td>52%</td>
<td>76%</td>
<td>47%</td>
<td>82%</td>
<td>87%</td>
<td>70%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>100%</td>
<td>100%</td>
<td>99%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>70%</td>
<td>-67%</td>
<td>61%</td>
<td>-22%</td>
<td>63%</td>
<td>22%</td>
<td>72%</td>
<td>90%</td>
<td>36%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>77%</td>
<td>25%</td>
<td>60%</td>
<td>-800%(^2)</td>
<td>78%</td>
<td>-320%(^3)</td>
<td>94%</td>
<td>100%</td>
<td>-86%</td>
</tr>
<tr>
<td>Pte. Claire + Jetty 1</td>
<td>12%</td>
<td>-33%</td>
<td>-3%</td>
<td>-113%</td>
<td>-2%</td>
<td>-19%</td>
<td>7%</td>
<td>-41%</td>
<td>-24%</td>
</tr>
<tr>
<td>Average (all points)</td>
<td>74%</td>
<td>42%</td>
<td>63%</td>
<td>-97%</td>
<td>69%</td>
<td>-23%</td>
<td>63%</td>
<td>85%</td>
<td>34%</td>
</tr>
<tr>
<td>Average (without Pte.</td>
<td>62%</td>
<td>26%</td>
<td>51%</td>
<td>-126%</td>
<td>57%</td>
<td>-27%</td>
<td>60%</td>
<td>63%</td>
<td>26%</td>
</tr>
</tbody>
</table>

1. % Improvement = \(100\times \frac{\text{Base Case extreme violation count} - \text{UW extreme violation count}}{\text{Base Case extreme violation count}}\)
2. Inflated due to very small number of extreme violation counts (18 for UW plan versus 2 in base case).
3. Inflated due to very small number of extreme violation counts (21 for UW plan versus 5 in base case).

3.2.5 Four-point control formulation including Montreal – version 2

Another optimization problem was formulated and solved in order to better assess if the overall management objective (improving/maintaining performance at all evaluation points, including Montreal, for all NBS scenarios) is really not achievable. The optimization formulation solved in Section 3.2.4 was slightly modified to generate a second 4-point UW plan (with Montreal) and this plan is referred to as the 4-point UW plan (with Montreal, v2). The optimization problem was exactly the same as the one solved in Section 3.2.4 except the soft constraints on Lake Superior levels described in Section 2.5.4 were eliminated in order to provide a maximum amount of flexibility in the hopes of trading off an increased range on Lake Superior (beyond the 182.76 - 183.86 m range in Section 2.5.4) for improvements relative to the base case downstream of Lake Superior.

Figure 12 compares the 4-point UW plan (with Montreal, v2) in the objective function and the 4-point UW plan (without Montreal) from Section 3.2.2. Note the new evaluation point at Montreal (Pointe Claire and Jetty1) is now in each of the subplots in Figure 12. Table 10 reports the improvement of the 4-point UW plan (with Montreal, v2) over base case (in percent) at different evaluation points under different NBS scenarios and can be compared directly with Table 7 and Table 9. Observations from Figure 12 and Table 10 can be summarized as follows:

- Similar performance at Montreal is observed for the 4-point UW plan (with Montreal, v2) and the 4-point UW plan (with Montreal) in Section 3.2.4. The 4-point UW plan (with Montreal, v2) generates performance at Montreal that is often degraded relative to the base case (Scenarios 2, 4, 6, 8). It is again interesting to note that Montreal performance
is equal or better than the base case in all of the low NBS scenarios (1, 3, 5, 7) but not the high NBS scenarios (2, 4, 6).

- Managing for Montreal using only the upstream four control points (i.e. assuming no structures downstream of Montreal would be built) causes substantial degradation in UW plan performance at upstream evaluation points (average of 40% improvement compared to 68% in Table 7 for evaluation points upstream of Montreal).

- Most degradation of upstream performance after considering Montreal in the objective is observed at Lake MH and Lake St. Clair. This observation might be solution specific, and there may be other solutions to the same optimization problem being less affected by considering Montreal.

- The 4-point UW plan (with Montreal, v2) still improves upon base case in all evaluation points upstream of Lake Ontario in all scenarios.

When Montreal is considered as an evaluation point in the objective function, no solution has been identified that meets the overall objective of improving performance at all evaluation points for all NBS scenarios. The best evidence of this is the histograms describing the magnitude and frequency of various extreme water levels simulated under the entire 50,000 year LOSL stochastic NBS data set shown in Figure 13. Relative to the base case plan Figure 13 shows that, although new extreme levels would be infrequent in 4-point UW plan with Montreal (0.51% of the time), the frequency and the magnitude of extreme levels beyond the range simulated for the historical period would be increased. Including the lower St. Lawrence as an evaluation point in the frequency based objective function cannot improve the performance of this part of the system compared to the base case.
Figure 12- Performance the 4-point UW plan (with Montreal, v2) and 4-point UW plan (without Montreal) for the eight NBS scenarios in comparison with base case.
Figure 13. Histograms of monthly average extreme violation magnitudes for Montreal harbour for the 50,000 year simulation under the 4-point UW (with Montreal) and base case plans – total number of time intervals (months) is 600,000. Note that although Montreal Harbour is included in the objective function, base case performance is still better than optimized UW plan.

Table 10: Improvement\(^1\) of 4-point UW plan (with Montreal, v2) at each evaluation point for each NBS scenario relative to base case.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>75%</td>
<td>65%</td>
<td>60%</td>
<td>63%</td>
<td>53%</td>
<td>44%</td>
<td>100%</td>
<td>96%</td>
<td>70%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>41%</td>
<td>70%</td>
<td>24%</td>
<td>75%</td>
<td>29%</td>
<td>34%</td>
<td>12%</td>
<td>42%</td>
<td>41%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>56%</td>
<td>43%</td>
<td>62%</td>
<td>32%</td>
<td>57%</td>
<td>32%</td>
<td>65%</td>
<td>69%</td>
<td>52%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>81%</td>
<td>100%</td>
<td>81%</td>
<td>100%</td>
<td>75%</td>
<td>100%</td>
<td>88%</td>
<td>100%</td>
<td>91%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>53%</td>
<td>-117(^2)</td>
<td>53%</td>
<td>-39%</td>
<td>30%</td>
<td>20%</td>
<td>51%</td>
<td>50%</td>
<td>13%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>38%</td>
<td>25%</td>
<td>27%</td>
<td>-350(^2)</td>
<td>20%</td>
<td>-140(^2)</td>
<td>58%</td>
<td>100%</td>
<td>-28%</td>
</tr>
<tr>
<td>Pte. Claire + Jetty 1</td>
<td>11%</td>
<td>-67%</td>
<td>2%</td>
<td>-213(^2)</td>
<td>2%</td>
<td>-38%</td>
<td>-4%</td>
<td>-68%</td>
<td>-47%</td>
</tr>
<tr>
<td>Average (all points)</td>
<td>46%</td>
<td>9%</td>
<td>41%</td>
<td>-66%</td>
<td>36%</td>
<td>1%</td>
<td>45%</td>
<td>49%</td>
<td>27%</td>
</tr>
<tr>
<td>Average (without Pte. Claire + Jetty 1)</td>
<td>57%</td>
<td>31%</td>
<td>51%</td>
<td>-20%</td>
<td>44%</td>
<td>15%</td>
<td>62%</td>
<td>76%</td>
<td>40%</td>
</tr>
</tbody>
</table>

1. % Improvement = \(\frac{100\text{[Base Case}_{\text{extreme violation count}} - \text{UW}_{\text{extreme violation count}}]}{\text{Base Case}_{\text{extreme violation count}}}\)
2. Inflated due to very small number of extreme violation counts in base case.

3.3 Multi-objective optimization results for frequency-based objective and regulation costs associated with St. Clair and Niagara River control points

The 4-point UW plans (with and without Montreal in the objective function) described in Sections 3.2.2 and 3.2.4 were optimized for performance only and ignore costs. As such, these plans estimate the best possible performance improvements associated with two new control structures on the Great Lakes but are not at all cost-effective. Ignoring cost implications is not at all realistic and thus this section deals with a multi-objective formulation that considers costs.
Four-point control plans were optimized as a multi-objective (bi-objective) optimization problem to minimize the frequency-based objective and minimize the regulation cost objective. This optimization approximates the tradeoff between improved system performance (frequency-based objective) and regulation costs. The only difference between the single-objective optimization problems solved in Sections 3.2.2 and 3.2.4 and the multi-objective optimization models below is the addition of Equation 22 as a cost objective (structure + dredging costs) as described in Section 2.5.5. All multi-objective optimization problems in this report were solved using the PA-DDS algorithm as described in Section 2.6.

This bi-objective optimization problem is solved using two versions of the frequency based objective function. In the first version, only six evaluation points (Lake Superior, Lake MH, Lake St. Clair, Lake Erie, Lake Ontario, and Iroquois + Saunders headwaters) are considered in the frequency based objective function. Note that the Iroquois and Saunders locations are combined to define one evaluation point with the same weight or importance as each upstream lake. With this approach, we roughly assume that structures could be built downstream of Montreal to completely mitigate water level extremes on the Lower St. Lawrence. In the second version of the frequency based objective function, the evaluation point associated with the Lower St. Lawrence River at Montreal (Pointe Claire + Jetty1) was also included in the objective function to evaluate the extent to which extreme levels at Montreal might be mitigated by control structures only upstream of Montreal and highlights the associated impacts on performance at upstream evaluation points. Note that the Pointe Claire and Jetty1 locations are combined to define one evaluation point with the same weight or importance as each upstream lake. In other words, it is assumed that no control structures would be added to the lower St. Lawrence to control water levels at Montreal.

3.3.1 Bi-Objective Optimization without Montreal in Frequency-Based Objective

PA-DDS was applied iteratively to solve the bi-objective problem with the first version of the frequency base objective function in order to continually improve the tradeoff approximation and focus the algorithm on the most relevant parts of the tradeoff. The PA-DDS algorithm was initially seeded with the results of multiple single objective optimization trials (different solutions) from the analyses of Section 3.2.2. Results from the first PA-DDS iterates showed two distinct clusters of solutions on the tradeoff. One cluster with overall best frequency-based objective function value (approximately -20) but almost $30 billion in cost and another cluster with reasonable cost (around $5-6 billion) but very large frequency based objective function value (more than 1000). Additional iterates demonstrated that the frequency-based objective of the second cluster of solutions could be substantially reduced without any noticeable increase in their cost. Therefore, a further suite of experiments was designed to sequentially solve the problem while focusing on only these two clusters of solutions and avoiding altogether the evaluation of solutions with poor frequency-based objective values solutions and thus costs lower than $5 billion. In these final experiments, a multi-objective pre-emption concept similar to that in Razavi et al. (2009) is applied to reject the complete evaluation of solutions with an unacceptably poor frequency-based objective function. This requires defining a threshold for
rejecting the complete evaluation of solutions based on the frequency objective and the initial iterates used a value of 1000 for this threshold. The threshold was reduced sequentially in later iterations to a final value of 0.

Figure 14 represents the tradeoff between regulation costs and the first version of frequency based objective function (Lower St. Lawrence River at Montreal evaluation points not considered). This tradeoff is the aggregated result of the iterative PA-DDS approach described above (more than ten sequential iterates, as well as multiple independent trials of PA-DDS, totaling more than 100,000 evaluated solutions). Each point on this figure represents the two objective function values for each alternative regulation plan. Two solutions (highlighted in blue and green) from this tradeoff are the most interesting ones to be evaluated more. The $29 billion solution (highlighted in blue) in Figure 14 clearly uses extrapolated excavation costs beyond the data in Table 2 and Table 3. This cost is controlled primarily by dredging costs (more than 95%) driven by maximum differences in flows between the 4-point $29 billion UW plan actual release and the current conveyance regime of 9450 m$^3$/s for the St. Clair River and 6520 m$^3$/s for the Niagara River flow (differences based on lakes at same levels – see Section 2.5.5). The solution also has a very good negative frequency-based objective value. For the 4-point $6 billion UW plan, the maximum increase in flow over the current conveyance regime is 1751 m$^3$/s for the St. Clair River and 5467 m$^3$/s for the Niagara River flow.

The tradeoff in Figure 14 shows a substantial discontinuity at a frequency based objective of -20. There are at least two potential causes of this discontinuity. First of all, the PADDs algorithm

![Figure 14. Estimated tradeoff between the frequency-based objective function (Equation 21) and the regulation costs associated with controlling flow on the Niagara and St. Clair Rivers (Equation 22). Note that a negative value of the frequency-based objective implies that the frequency of violating extremes is improved or equal to the base case everywhere and for all eight NBS scenarios. The blue and green coloured solutions are compared in Figure 15.](image-url)
only approximates the true set of tradeoff solutions and there is no guarantee the algorithm will find all tradeoff solutions (in particular with such a high dimensional multi-objective optimization problem). As such, many solutions could exist that span this discontinuity and simply have not been discovered by the algorithm. In addition, the nature of the optimization problem could be the second cause of this discontinuity. The extreme increase in costs is driven by the need to excavate drastically more material in order to pass required flows down the St. Clair River. Compare the maximum increases in St. Clair River flow over the current conveyance regime for the green (1751 m³/s) and blue (6520 m³/s) solutions. Note the maximum increases on the Niagara are much closer (5467 m³/s versus 6520 m³/s). The form of the frequency based objective function (Equation 21), which focuses on improving the worst performing NBS scenario relative to the base case, is important in understanding the discontinuity. Equation 21 implies that for a given lake, performance is measured/controlled by performance in only the critical NBS scenario and the non-critical performance in the other seven NBS scenarios is ignored. Thus, as the frequency based objective is improved, there will be points (i.e. the discontinuity) at which improving the objective can only accomplished by improving performance in a new critical NBS scenario. Improved performance in such a new critical NBS scenario could require drastically more dredging (large increase in costs) before a corresponding improvement in the frequency-based performance measure is observed.

Figure 15 compares the frequency or risk of going beyond historical simulated extremes for the two tradeoff solutions ($6 and $29 billion) highlighted in Figure 14. Figure 15 shows that in general, the more expensive solution improves overall performance. However, there are a few cases where the less expensive plan does better than the more expensive plan (e.g. Lake Superior and Lake St. Clair in Scenario 4).
Figure 15 - Performance of the $29 billion (blue) and $6 billion (green) 4-point UW plans from Figure 14 for the eight NBS scenarios in comparison with base case.
Figure 16 compares the %Improvement measures averaged over all evaluation points for the $29 billion and $6 billion 4-point UW plans from Figure 13. Although the more expensive plan generates improved performance over the less expensive plan at almost all evaluation points (not Lake Superior), the plan performance levels are fairly comparable. The average %Improvement across all evaluation point and NBS scenarios for the $6 billion solution is 59% compared to 71% for the $29 billion solution.

![Graph showing average %Improvement at each evaluation point for $29 billion and $6 billion 4-point UW plans](image)

**Figure 16 – Average %Improvement at each evaluation point of the $29 billion (blue) and $6 billion (green) 4-point UW plans from Figure 14 relative to the base case (averaged across eight NBS scenarios).**

3.3.2 Bi-Objective Optimization with Montreal in Frequency-Based Objective

Another bi-objective optimization trial is solved to approximate the tradeoff between the regulation cost and the frequency based objective function with 4 control structures when including lower St. Lawrence (Lac St. Louis at Pointe Claire and Montreal harbour at Jetty1) in the frequency based objective function. This bi-objective problem is solved with approximately the same computational budget and iterative approach for the bi-objective optimization explained in section 3.3.1. Figure 17 compares the tradeoff between the frequency based objective function and the regulation cost of four control structures when including or ignoring the lower St. Lawrence in the frequency based objective function.
Comparing the two tradeoffs (green and black points) shows that considering Montreal in the frequency based objective function does not let us improve over the base case in all the evaluation points, and hence the best frequency based objective function on the black series is positive for all solutions. Moreover, at each level of cost, the frequency based objective function of the black series is much worse than the corresponding one on the green series. However, a large portion of this degradation is due to the violations in lower St. Lawrence evaluation points. For two interesting solutions on the black series the violations in lower St. Lawrence evaluation points is subtracted from the frequency based objective function to get the blue and red points. Comparing each of the blue or red points to the same costly points on the green series represents how much the frequency based objective function upstream of Lac St. Louis at Pointe Claire is degraded because of considering lower St. Lawrence surface elevation in the frequency based objective function. This value is almost 120 units for the red point and 180 units for the blue point, both of which amount to nearly 50% of objective function degradation.

When Montreal is considered as an evaluation point in the frequency based objective function of this bi-objective analysis, no solution has been identified that meets the overall objective of improving performance at all evaluation points for all NBS scenarios. Explicit evidence of this is shown in Figure 17 which evaluates the best performing tradeoff solution in terms of the frequency based objective (referred to as the $8 billion 4-point UW plan (with Montreal) solution). Figure 17 is two histograms describing the magnitude and frequency of various extreme water levels for the base case and $8 billion 4-point UW (with Montreal) plans as simulated with the entire 50,000 year LOSL stochastic NBS data set. Relative to the base case plan, Figure 17 shows that the frequency and the magnitude of extreme levels beyond the range simulated for the historical period would be increased under the $8 billion 4-point UW plan.
(with Montreal). Including the lower St. Lawrence as an evaluation point in the frequency based objective function cannot improve the performance of this part of the system compared to the base case.

![Figure 18. Histograms of monthly average extreme violation magnitudes for Montreal harbour for the 50,000 year simulation under the S8 billion 4-point UW (with Montreal) and base case plans – total number of time intervals (months) is 600,000. Note that although Montreal Harbour is included in the objective function, base case performance is still better than optimized UW plan.](image)

### 3.4 Summary of Findings for Managing Montreal Water Levels

Under the four regulation structures analyzed in this report, a single regulation policy was not found to improve performance everywhere in the system for all NBS scenarios and that was due to the inclusion of Montreal as an evaluation point. Three different optimization problems were solved with Montreal as an evaluation point (see Sections 3.2.4, 3.2.5 and 3.3.2), and each of them were solved multiple times, to try and achieve this goal. These results suggest to us that a solution likely does not exist that improves performance everywhere in the system for all NBS scenarios. However, given the number of decision variables (78) and non-linearity of this problem, it is possible that our solutions are poor approximates of the global minimum or the true tradeoff and thus there is the potential that there are solutions that would improve performance everywhere in the system for all NBS scenarios even with Montreal.

In preliminary analyses, we optimized the four control point system on each NBS scenario individually (single objective optimization problem with Equation 20 as the objective function) with the levels at Montreal Harbour in the frequency-based objective function. Note that these preliminary analyses were conducted under a preliminary suite of assumptions that were slightly inconsistent with the assumptions reported on in this report. When optimized on each of the eight scenarios individually, the eight different 4-point control plans were capable of improving the system performance everywhere, including Montreal Harbour, over the base case. When optimized on a single scenario, the improvement at Great Lakes evaluation points (i.e., Lake Superior through Lake Ontario) was substantial such that for most scenarios the violation frequency of the 4-point UW plans at these evaluation points was zero or much lower than the base case. For the Montreal Harbour evaluation point, the 4-point UW plans when optimized on one single scenario also outperformed the base case, but the extent of improvement was
considerably less compared to the improvement observed for the Great Lakes evaluation points. Therefore, if NBS conditions could be forecasted, there is a reasonable chance the system could be managed adaptively (switch between NBS forecast based rule curves) to achieve less frequent extreme lake levels relative to the base case for all evaluation points and all NBS scenarios.

Under the suite of plans developed using four control structures in this analysis, including those developed with improved performance at Montreal as an objective, Montreal water levels were observed to degrade relative to the base case. Since operating the two additional structures on the St. Clair and Niagara Rivers as described in any of the above UW plans would have a detrimental impact on the lower St. Lawrence River, the Boundary Waters Treaty (http://www.ijc.org/rel/agree/water.html) would suggest that if such a plan were to be approved, that the IJC would require, as a condition of its approval, that “suitable and adequate provision” be made to protect interests in the lower St. Lawrence River. This may require additional structures installed on the lower St. Lawrence to avoid degraded levels at Montreal and perhaps even further downstream. The original scope of this report did not include an assessment of whether such additional structures on the Lower St. Lawrence would be necessary. Therefore, further optimization experiments should be conducted to determine how much improvement on the Upper Great Lakes is possible with the suite of four control structures but without degrading Montreal performance.

With the above implications in mind, a more detailed evaluation of any of the UW plans (e.g., as in the following section) should focus on those plans that do not include Montreal in the objective function, in order to achieve the greatest benefits upstream. Such UW plans provide an estimate of the upper bound of performance improvements that are possible upstream of Montreal and in the worst case, any such UW plan might require additional structures installed on the lower St. Lawrence. If the IUGLS Study Board deems the upper bound performance improvements to be substantial or significant enough, and their corresponding cost estimates are not impossibly large, then the further optimization experiments described above should be carried out.

3.5 Further Performance Assessment of the $6 billion 4pt UW-Plan (without Montreal)

There are dozens of 4-pt UW plans (without Montreal) with varying costs depicted in Figure 14 and fully evaluating additional aspects of their performance is beyond the scope of this report. However, one solution potentially interesting to decision-makers is the $6 billion 4-point UW plan (without Montreal) given the incredibly smaller cost and relatively small decrease in performance relative to the most effective $29 billion 4-point UW plan (without Montreal). Therefore, this section aims to highlight other aspects or impacts of the $6 billion 4-point UW plan (without Montreal) in Sections 3.5.1, 3.5.2 and 3.5.2 below. For brevity, the $6 billion 4-point UW plan (without Montreal) will be referred to as the UW plan in this Section but will retain full nomenclature details in the Figures and Tables.
3.5.1 Performance over the Multi-Scenarios

Figure 15 presented earlier shows the comparative results of the UW plan optimized through the multi-NBS scenario optimization framework over the eight NBS scenarios. Note that the frequency of going beyond the historical simulated extremes is also called the frequency of violation in this report and this frequency is the ratio of the number of months when the average level goes beyond the monthly historical simulated extremes over the total number of months in that scenario. As can be seen in Figure 15, the UW plan substantially outperforms the base case at all evaluation points and in all scenarios. Table 11 below also reports the improvement of UW plan over base case (in percent) at different evaluation points under different NBS scenarios.

Table 11- Improvement\(^1\) of $6 billion 4--point UW plan (without Montreal) at each evaluation point for each NBS scenario relative to base case

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>84%</td>
<td>78%</td>
<td>81%</td>
<td>74%</td>
<td>100%</td>
<td>59%</td>
<td>100%</td>
<td>100%</td>
<td>85%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>52%</td>
<td>57%</td>
<td>65%</td>
<td>10%</td>
<td>46%</td>
<td>15%</td>
<td>75%</td>
<td>68%</td>
<td>48%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>49%</td>
<td>36%</td>
<td>65%</td>
<td>91%</td>
<td>42%</td>
<td>63%</td>
<td>65%</td>
<td>49%</td>
<td>58%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>97%</td>
<td>89%</td>
<td>98%</td>
<td>100%</td>
<td>97%</td>
<td>87%</td>
<td>100%</td>
<td>100%</td>
<td>96%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>61%</td>
<td>-17%(^2)</td>
<td>53%</td>
<td>0%</td>
<td>22%</td>
<td>4%</td>
<td>65%</td>
<td>40%</td>
<td>28%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>48%</td>
<td>75%</td>
<td>32%</td>
<td>0%</td>
<td>-2%(^3)</td>
<td>-20%(^4)</td>
<td>88%</td>
<td>100%</td>
<td>40%</td>
</tr>
<tr>
<td>Avg.</td>
<td>65%</td>
<td>53%</td>
<td>66%</td>
<td>46%</td>
<td>51%</td>
<td>35%</td>
<td>82%</td>
<td>76%</td>
<td>59%</td>
</tr>
</tbody>
</table>

1. % Improvement = $\frac{100 \times (\text{Base Case extreme violation count} - \text{UW extreme violation count})}{\text{Base Case extreme violation count}}$
2. The number of violations in base case is 6 (frequency=0.71%) while the same number in UW plan is 7 (frequency=0.83%).
3. The number of violations in base case is 42 (frequency=4.7%) while the same number in UW plan is 43 (frequency=4.8%).
4. The number of violations in base case is 5 (frequency=0.52%) while the same number in UW plan is 6 (frequency=0.63%).

Overall observations from Figure 15 and Table 11 can be summarized as follows:

- Overall improvement (averaged) at all evaluation points and in all NBS scenarios is 59% for the UW plan relative to the base case.
- Improvement at Lake Erie is 96% suggesting that the UW plan can almost completely keep Lake Erie level within the range of extremes simulated for the historical period.
- Improvement at Lake Ontario is the smallest of all evaluation points (28% averaged over all NBS scenarios).
- Strictly speaking, this UW plan does not meet the objective of improving performance at all evaluation points for all NBS scenarios (see notes 2, 3 and 4 under Table 11). However, in comparison with the base case, this plan generates only one additional month (out of 840 in Scenario 2 or 960 in Scenario 5) where each evaluation point has a level beyond their respective simulated historical extremes. Therefore, for all intents and purposes, this 4-point UW plan does meet the objective of improving/maintaining
performance at all evaluation points for all NBS scenarios. In fact, validation performance assessed in the next section demonstrates improved overall performance on Lake Ontario and the IHW+SHW evaluation points over the full 50,000 year simulation.

3.5.2 Validation Performance of $6 billion 4-point UW-Plan and Base Case over the full 50,000 year LOSL NBS sequence

The base case and UW plan were simulated for the entire historical period with initial lake levels set to the year 1900 levels. Statistics on plan performance with regards to the frequency of violating monthly extreme lake levels, the magnitudes of these violations, the distribution of lake levels and the distribution connecting channel flows were analyzed. These are summarized in the following Tables and Figures.

Table 12 reports the frequency of violating extremes (upper and lower) as well as the frequencies associated to lower extreme and upper extreme separately. The total violation frequencies are substantially lower for the 4pt UW-Plan compared to the base case.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>UW Plan Frequency of Violating Extremes</th>
<th>Base Case Frequency of Violating Extremes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>0.27%</td>
<td>2.31%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>0.79%</td>
<td>2.59%</td>
</tr>
<tr>
<td>Lake St Clair</td>
<td>1.19%</td>
<td>2.75%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>0.03%</td>
<td>1.98%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>0.42%</td>
<td>0.87%</td>
</tr>
<tr>
<td>IHW</td>
<td>0.08%</td>
<td>0.64%</td>
</tr>
<tr>
<td>SHW</td>
<td>0.04%</td>
<td>0.03%</td>
</tr>
<tr>
<td>All 7 above evaluation points</td>
<td>3.8%</td>
<td>6.2%</td>
</tr>
<tr>
<td>All 7 evaluation pts except Lake St Clair</td>
<td>1.0%</td>
<td>5.5%</td>
</tr>
</tbody>
</table>

1. Consider violation of extremes to be a violation at one or more evaluation points (e.g. at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes).

Figure 19 and Figure 20 show the histograms of the magnitude of extreme violations at all evaluation points. As an example, Lake Superior when operated through $6 billion 4pt UW-Plan goes beyond the upper extremes (see upper left subplot in Figure 19) by 0-0.05 metres in 875 of the 600,000 (50,000 * 12) months in the simulation, while in the base case it goes beyond the
upper extreme by 0-0.05 metres in 3567 months. For SHW, since we did not assume an upper extreme as discussed in Section 2.5.1, this figure only has the histogram plot for SHW when the water level violates lower extreme. Figure 19 and Figure 20 show the $6 billion 4pt UW-Plan generates substantially lower numbers in each bin for most evaluation points. For example, the 4pt UW-Plan generates very few violations for Lake Erie, Lake Superior and IHW. Two exceptions are for Lake Michigan-Huron when violating the upper extreme and Lake Ontario when violating the upper extreme and in both these exceptions, the 4pt UW-Plan and the base case failure frequencies and magnitudes are quite similar. In the only other exception at SHW, the 4pt UW-Plan generates slightly higher failure frequencies and higher failure magnitudes. Overall, these histograms show that in addition to reducing the frequency of extreme violations, the magnitudes of the extreme violations are also substantially reduced by the UW plan (despite the fact that these magnitudes are not considered in the optimization formulation).

Figure 21 compares the UW-Plan to the base case in terms of the percentiles of water levels at all evaluation points (Lake Superior through Saunders headwaters) while Figure 22 compares the 4pt UW-Plan to the base case in terms of the percentiles of connecting channel flows (St. Marys River through Lake Ontario outflow). In Figure 21, the UW-Plan clearly compresses the range of water levels simulated over the 50,000 year period for all evaluation points except for Saunders headwater. In this exception, the range of lake levels are approximately the same in the UW-Plan and the base case. Figure 22 shows that the compressed water level ranges in the UW plan appear to require expanding the range of flows experienced in the connecting channels (except for Lake Ontario outflow).
Figure 19. Histograms of monthly average extreme violation magnitudes for Lake Superior through Lake Erie evaluation points for the 50,000 year simulation under the $6 billion 4–point UW plan (without Montreal) and base case plans – total number of time intervals (months) is 600,000.
Figure 20. Histograms of monthly average extreme violation magnitudes for Lake Ontario through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the $6 billion 4–point UW plan (without Montreal) and base case plans – total number of time intervals (months) is 600,000.
Figure 21. Select percentiles from the frequency distribution of monthly average water levels for Lake Superior through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the $6 billion 4-point UW plan (without Montreal) and base case plans.
Figure 22. Select percentiles from the frequency distribution of monthly average connecting channel flows for the St Marys River through Lake Ontario Outflow for the 50,000 year simulation under the $6 billion 4--point UW plan (without Montreal) and base case plans. Note that there is no structure controlling Lake St. Clair (Detroit River) outflows in the UW plan.

The longest periods when water levels at each evaluation point are lower than the historical minimum extreme and higher than the historical maximum are calculated for the base case and UW plan for the 50,000 year simulation in Table 13. In general, the Table 13 shows that the extended periods of extreme levels are reduced under the UW plan in comparison to the base case (even for levels at Pointe Claire and Jetty 1 in Montreal which were not included in the objective function). The most substantial improvements of UW plan over the base case are on Lake Superior (longest period of extended high levels now 30 months instead of 109), Lake Michigan-Huron (longest period of extended low levels now 67 months instead of 169) and Lake Erie (longest period of extended low levels now 4 months instead of 90).
Table 13. Longest periods (measured in the number of months) in 50,000 years when water surface elevation at each evaluation point is lower than the minimum extreme and higher than the maximum extreme. $6 billion 4-point UW plan (without Montreal) compared to the base case.

<table>
<thead>
<tr>
<th></th>
<th>Extreme violated</th>
<th>SP</th>
<th>MH</th>
<th>SC</th>
<th>ER</th>
<th>ON</th>
<th>IHW</th>
<th>SHW</th>
<th>Pointe Claire</th>
<th>Jetty1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Base Case</td>
<td>min</td>
<td>45</td>
<td>169</td>
<td>98</td>
<td>97</td>
<td>65</td>
<td>23</td>
<td>5</td>
<td>20</td>
<td>54</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>109</td>
<td>63</td>
<td>20</td>
<td>21</td>
<td>19</td>
<td>7</td>
<td>0</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>$6 billion 4pt UW-Plan (without Montreal)</td>
<td>min</td>
<td>8</td>
<td>67</td>
<td>44</td>
<td>4</td>
<td>60</td>
<td>49a</td>
<td>11a</td>
<td>11</td>
<td>12</td>
</tr>
<tr>
<td></td>
<td>max</td>
<td>30</td>
<td>60</td>
<td>25a</td>
<td>9</td>
<td>32a</td>
<td>11a</td>
<td>0</td>
<td>5</td>
<td>5</td>
</tr>
</tbody>
</table>

a. Highlighted values are worse than base case.

3.5.3 Time Series Analysis of $6 billion 4-point UW Plan and Base Case

The following figures represent the time series of monthly average lake levels (Figure 23) and outflows (Figure 24) for the UW plan (blue) and Base Case (red) in NBS Scenario 2. These time series are just meant to show the similarity of the variation and range of levels over time between the UW and base case plans. For example, the Lake Superior outflows under the UW plan vary more erratically than the outflows under the base case in Figure 24 and this is due to the wide range of outflows deemed allowable under the UW plan in comparison to the base case. Lake Michigan-Huron outflows under the UW plan in Figure 24 look fairly similar in range and variation to the base case.
Figure 23. Time series of lake levels (Superior through Erie) under NBS scenario 2 for the $6 billion 4--point UW (without Montreal) and base case plans. UW plan (blue) and Base Case (red).
Figure 24. Time series of outflows from Lakes Superior through Ontario under NBS scenario 2 for the $6 billion 4-point UW (without Montreal) and base case plans. UW plan (blue) and Base Case (red).
4 Conclusions and Future Work

The current or base case Great Lakes regulation strategy will result in very frequent extreme lake levels, for extended periods of time, under extreme future climate scenarios. Extreme lake levels in this report imply lake levels that are beyond the range of levels simulated for the 1900-2008 historical period. For example, Lake Michigan-Huron and Lake St. Clair are simulated to experience frequencies of new extreme levels for between 20%-30% of all months simulated in multiple extreme NBS scenarios. Even in an extended 50,000 year simulation driven with a stochastic NBS data set built from the characteristics of the historical period NBS, for 6.4% of that simulation period, water levels for at least one evaluation point in the Great Lakes – Montreal system were simulated beyond the historical period simulated extremes. During this extended simulation, Lake Michigan-Huron is simulated to experience new extreme low levels continuously over a 14 year period. Optimizing two point multi-lake regulation plans using the existing structures controlling Lake Superior and Lake Ontario outflow are not capable of significant improvements over the base case regulation strategy in terms of improving performance under extreme climates.

Four point multi-lake regulation (Lakes Superior, Michigan-Huron, Erie and Ontario outflows are regulated) on the Great Lakes can simultaneously reduce (or maintain) the frequency of extreme water levels and reduce the magnitude of extreme violations across multiple extreme NBS scenarios and at all evaluation points in the system relative to the base case regulation strategy. This finding is specific to the Lake Superior through Upper St. Lawrence evaluation points and assumes structures are constructed on the Lower St. Lawrence that can completely mitigate extreme water levels in the Lower St. Lawrence River. For example, the frequency of experiencing new extreme lake levels under the base case regulation strategy for the eight extreme NBS scenarios are reduced by an average of 68% (averaged over all evaluation point – NBS scenario combinations). The cost of such a plan is extremely prohibitive (estimated at $29 billion dollars and almost completely due to estimated dredging costs). An alternative $6 billion dollar plan was identified that performs comparably well that reduces the frequency of new extreme water levels by an average of 59%. However, strictly speaking, the $6 billion dollar plan slightly degrades performance at the Saunders Headwater evaluation point in comparison with the base case. Dozens of other four-point multi-lake regulation plans were identified with a range of costs (and performance) between these $6 billion and $29 billion plans.

It is important to note that the cost estimates are very approximate and they ignore other costs that would be incurred with two new structures on the Great Lakes (on St. Clair and Niagara Rivers). First of all, many cost estimates for the above range of solutions are based on extrapolating outside of the range of data used to estimate these costs. Secondly, the cost estimates do not consider costs associated with building a lock system along the St. Clair River that would be required to move ships around any regulation structure on the St. Clair River. Lastly, the cost estimates ignore mitigation costs that may be required downstream on the Lower St. Lawrence River. Although such costs are not evaluated in this report, if St. Lawrence mitigation was required, these costs would easily be into the billions of dollars based on the
Levels Reference Study (1993) cost estimates. Given the approximate nature of the regulation cost estimates, further study on multi-lake regulation should give careful consideration to revising and updating these cost estimates for changes to the St. Clair River and/or Niagara River channels as well as mitigation costs on the lower St. Lawrence River.

Under the four regulation structures analyzed in this report, a single multi-lake regulation policy was not found that improves performance everywhere in the system (from Lake Superior down to Montreal) for all NBS scenarios and that was due to the inclusion of Montreal as an evaluation point. The original scope of this report did not include an assessment of whether such additional structures on the Lower St. Lawrence would be necessary. The multi-lake plans described above provide an estimate of the upper bound of performance improvements that are possible upstream of Montreal and in the worst case, any such plan might require additional structures installed on the Lower St. Lawrence. If the IUGLS Study Board decides that, based on the multi-lake plan performance and estimates of their costs presented here, further study of multi-lake regulation is warranted, it is recommended that the additional optimization experiments described below be carried out as part of this analysis. For example, the optimization formulation could be modified to give the Montreal evaluation point more weight in the objective function such that it is essentially a constraint in the optimization formulation.

The following list summarizes all other suggested additional analyses that should be considered in any future work:

- Determine the performance improvements achievable due to only one new structure on the Niagara River. Repeat for only one new structure on the St. Clair River.
- Expend additional computational efforts to improve solution quality through further optimization experiments. Better solutions almost certainly exist but as more and more search is conducted, the probability that a near-optimal solution has been achieved increases. High performance computing facilities should be considered in any future work and should definitely be utilized for any future work that is seeking to recommend a new 3- or 4-point multi-lake regulation strategy for implementation.
- Attempt to generate alternate sets of rule curves for forecasted NBS or for extreme lake level conditions so that there was even more adaptability to the multi-lake rule curves.
- Some consideration should be given to alternative objective functions to those in this report. This should begin with stakeholders and/or the Study Board carefully examining all aspects of one of the UW plans to identify undesirable performance characteristics.
- Given the empirical nature of the St. Lawrence simulation equations used in this report, and their assumed decrease in simulation accuracy relative to upstream lake level simulation, future regulation studies beyond the exploratory level requiring simulation of this part of the system should strongly consider re-evaluating how well the St. Lawrence simulation equations predict Montreal water levels. In particular, this evaluation could be conducted for the years that have passed since the empirical equations were last fitted to the data.
5 References


6 Appendices

6.1 Appendix 1 – Routing equations.

Set of routing equations to solve for determining Lake Michigan-Huron through Lake Erie water levels and lake outflows.

\[
NBS_3+D_3-QR_2-QO_3+k_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{b_2} = \\
\{ -0.5k_2b_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{(b_2-1)} - 0.5k_2a_2\varphi(\varphi z_2+(1-\varphi)z_3-ym_2)^{(a_2-1)}(z_2-z_3)^{b_2} \} \Delta Z_2 \\
+ \\
\{ 0.5k_2b_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{(b_2-1)} - 0.5k_2a_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{(a_2-1)}(z_2-z_3)^{b_2} + s a_3/\Delta t \} \Delta Z_3
\]

\[
NBS_2+D_2+QR_2+QO_1-k_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{b_2} = \\
\{ 0.5k_2b_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{(b_2-1)} + 0.5k_2a_2\varphi(\varphi z_2+(1-\varphi)z_3-ym_2)^{(a_2-1)}(z_2-z_3)^{b_2} + sa_2/\Delta t \} \Delta Z_2 \\
+ \\
\{ -0.5k_2b_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{a_2}(z_2-z_3)^{(b_2-1)} + 0.5k_2a_2(\varphi z_2+(1-\varphi)z_3-ym_2)^{(a_2-1)}(z_2-z_3)^{b_2} \} \Delta Z_3
\]

Subscripts:
1: Lakes Michigan-Huron
2: Lake St. Clair
3: Lake Erie

Variables:
NBS: Net Basin Supply
QO: Lake outflow
QR: Ice retardation
D: Diversion (+ into the lake, - out of the lake)
A: Lake surface area
Z: Lake elevation
\(\Delta t\): model timestep
\(\varphi\): weighting coefficient (between 0 and 1) for stage-discharge relationship
K: empirical coefficient for stage-discharge relationship
a: empirical coefficient for stage-discharge relationship
b: empirical coefficient for stage-discharge relationship
ym: empirical bottom elevation for stage-discharge relationship
\(\Delta Z\): change in lake surface elevation for \(\Delta t\)

MATLAB code simulating the St. Lawrence River system below

```matlab
function [SL] = StLawrence_simulation (ON, SL, year_num, month_num, qmonth_num)

Qont = ON.outflow.data((year_num-1)*4 + qmonth_num, month_num);
LevOnt = ON.level.qmean((year_num-1)*4 + qmonth_num, month_num);
```

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Nogd = SL.Iceogd.data((year_num-1)*4 + qmonth_num, month_num);
Ncar = SL.Icecar.data((year_num-1)*4 + qmonth_num, month_num);
Nihw = SL.Icihw.data((year_num-1)*4 + qmonth_num, month_num);
Nitw = SL.Icietw.data((year_num-1)*4 + qmonth_num, month_num);
Nmor = SL.Icemor.data((year_num-1)*4 + qmonth_num, month_num);
Nlsu = SL.Icelsu.data((year_num-1)*4 + qmonth_num, month_num);
Nshw = SL.Iceshw.data((year_num-1)*4 + qmonth_num, month_num);

LevKin = LevOnt - 0.03;
LevOgd = LevKin - Nogd * (Qont /(63.2800 * (LevKin - 62.4) ^ 2.0925)) ^ (1 / 0.4103);
LevCar = LevKin - Ncar * (Qont /(19.4908 * (LevKin - 62.4) ^ 2.3981)) ^ (1 / 0.4169);
LevIHW = LevKin - Nihw * (Qont /(23.6495 * (LevKin - 62.4) ^ 2.2886)) ^ (1 / 0.4118);
LevITW = LevKin - Nitw * (Qont /(24.2291 * (LevKin - 62.4) ^ 2.2721)) ^ (1 / 0.4118);
LevMor = LevKin - Nmor * (Qont /(23.9537 * (LevKin - 62.4) ^ 2.2450)) ^ (1 / 0.3999);
LevLSU = LevKin - Nlsu * (Qont /(21.6030 * (LevKin - 62.4) ^ 2.2866)) ^ (1 / 0.3749);

if LevSHW > 73.87
    LevLSU = LevSHW + 1408000 * Qont ^ 2.188 / (LevSHW - 55.0) ^ 12.638;
    LevMor = LevLSU + 6.79900 * Qont ^ 1.913 / (LevLSU - 55.0) ^ 6.9950;
    LevITW = LevLSU + 1.00000 * Qont ^ 1.841 / (LevLSU - 55.0) ^ 5.8910;
end

LevKin = SL.Kin.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevOgd = SL.Ogd.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevCar = SL.Car.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevIHW = SL.IHW.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevITW = SL.ITW.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevMor = SL.Mor.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevLSU = SL.Lsu.qlevel((year_num-1)*4 + qmonth_num, month_num);
LevSHW = SL.SHW.qlevel((year_num-1)*4 + qmonth_num, month_num);

Qcari = SL.Ottawa.data((year_num-1)*4 + qmonth_num, month_num);
Qchat = SL.Chateauguay.data((year_num-1)*4 + qmonth_num, month_num);
Qstfrn = SL.StFrancois.data((year_num-1)*4 + qmonth_num, month_num);
Qstmau = SL.StMaurice.data((year_num-1)*4 + qmonth_num, month_num);
Kmtl = SL.IceFactor.data((year_num-1)*4 + qmonth_num, month_num);
T = SL.tidal.data((year_num-1)*4 + qmonth_num, month_num);

[Qdpmi, Qstl, stlont] = Ottsplit (Qont, Qcari, Qchat);

Fstl = SL.Icepcl.data((year_num-1)*4 + qmonth_num, month_num);
Hpcl = 16.57 + ( Fstl * Qstl / 604.00) ^ 0.580;

SL.Qstl.data((year_num-1)*4 + qmonth_num, month_num) = Qstl;
SL.Pcl.qlevel((year_num-1)*4 + qmonth_num, month_num) = Hpcl;
SL.dpmi.data((year_num-1)*4 + qmonth_num, month_num) = Qdpmi;

Hmtl = jetty1(Qont, Qcari, Qchat, Qstfrn, Qstmau, Kmtl, T, Qdpmi, Qstl);

%%
function Hmtl = jetty1(Qont, Qcari, Qchat, Qstfrn, Qstmau, Kmtl, T, Qdpmi, Qstl)
%%
% Coded in Matlab by M. Asadzadeh Sept 2010
% This function calculates the level at Jetty 1 as a function of the
% followings:
% Qont : Lake Ontario outflow
% Qcari : Ottawa River flow at Cardillon
% Qchat : Chateauguay River flow
% Qstfrn : outflow of Saint-Francois River at Hemming Falls (02OF002)
% Qstmau : outflow of St Maurice River at La Gabelle (02NG005)
% Calculating:
% 1- Qdpmi : Des Praries plus Mille Iles river flows
% 2- Qstl : Lake St Louis outflow
% Calculating Level at Jetty1 (Montreal)
Hmtl = (Kmtl*(0.001757 * Qstl + 0.000684 * Qdpmi + 0.001161 * Qstfrn + 0.000483 * Qstmau)) ^ 0.6587 + 0.9392 * T;

function [Qdpmi, Qstl, stlont] = Ottsplit (Qont, Qcari, Qchat)

%% Re-Coded in Matlab by M. Asadzadeh Sept 2010 from the original FORTRAN

% Estimation of flows from Lake of Two Mountains and Lake St. Louis
% by Yin Fan and David Fay
% Flows are in m3/s and levels in m IGLD 1985
% Given flows from Lake Ontario, Ottawa River at Carillon, and Chateauguay River,
% the following relationships can be used to estimate the outflow from Lake St. Louis,
% and the Mille Iles and Des Prairies rivers (ie. the "backrivers").

% Qcari : Ottawa River Flow at Carillon
% Qont : Lake Ontario outflow
% Qchat : Chateauguay River flow
% Qvaud : Vaudreuil channel flow
% Qsta : Ste Anne channel flow
% Qdpmi : Des Praries plus Mille Iles river flows
% Qstl : Lake St Louis outflow

% First the flow split from Lake of Two Mountains into the Vaudreuil
% and Ste Anne channels must be calculated.
% This is complicated since there can be flow reversals in these
% channels when the Ontario
% outflow is very high relative to the Ottawa R outflow.
% For estimating the Vaudreuil Channel flow, one of three
% relationships are used depending on the conditions that follow.

% REAL*4 Qont, Qcari, Qchat, stlont, Qdpmi, Qv1, Qv2, Qv3, Qvaud
% REAL*4 Qsta4, Qsta5, Qstl, Qtemp

% Need to convert Ont flow and stlont flow to 10 cms for use in plans

% Qont = 10 * Qont;
% eqn (1)
Qv1 = 787.4 + 0.3479 * Qcari - 0.1336 * Qont - 0.9374 * Qchat;
% eqn (2)
Qv2 = -61 + 0.2975 * Qcari - 0.0356 * Qont + 16.77 * Qont / Qcari;
% eqn (3)
Qv3 = 24.2 + 0.295 * Qcari - 0.01885 * Qont - 0.0954 * (Qont - Qcari) * Qcari / Qont;

% Vaudreuil equation selection
% Note: negative Q vaud indicates flow from L St Louis
% into L Des Deux Montagnes)
if Qont <= 6600
  if Qcari <= 300.0
    Qvaud = 0.0;
  elseif Qcari > 300 && Qcari <= 900
    Qvaud = max(Qv3,0.0);
  else
    Qvaud = max(Qv2,0.0);
end
else
    if Qcari <= 300.0
        Qvaud = min(Qv1,0.0);
    else
        if (Qont - Qcari) > 7100.0 & (Qont - Qcari)*Qont/Qcari < 35000
            Qvaud = Qv1;
        else
            if Qcari < 900.0
                Qvaud = min(Qv1,Qv3);
            else
                Qvaud = min(Qv1,Qv2);
        end
    end
end
end

% For estimating the Ste. Anne Channel flow, two relationships are used
% depending on the conditions that follow.

% eqn (4)
Qsta4 = 585.4 + 0.2733 * Qcari - 0.05017 * Qont - 20.95 * Qont / Qcari;
% eqn (5)
Qsta5 = -125 + 0.2986 * Qcari + 0.02046 * Qont - 13.54 * Qont / Qcari;

% Ste Anne equation selection
if Qont <= 7800.0
    if Qcari <= 300.0
        Qsta = 0.0;
    else
        Qsta= max(Qsta5,0.);
    end
else
    if Qcari <= 300.0
        Qsta = Qsta4;
    else
        Qsta= min(Qsta4,Qsta5);
    end
end

% The flows from the Des Prairies and Mille Iles Rivers are then estimated by the
% following
% round to 10M3/S but in cms units
Qtemp = (107.2 + 0.7418 * Qcari - 0.6588 * Qvaud)/10.0; %
Qdpmi = round(Qtemp); %
Qdpmi=Qdpmi * 10.0; %

% The flow from Lake St. Louis is estimated using one of the following two equations
% depending on the magnitude of the Ontario outflow:
if Qont < 9000.0
    Qstl = -877.4 + 1.1652 * Qont + 4.367 * Qchat + 1.3956 * Qvaud + 0.8137 * Qsta;
else
    Qstl = 2751.0 + 2.091 * Qvaud + 0.7977 * Qont + 3.381 * Qchat;
end

% IN 10M3/S
Qont = Qont / 10.0 ;
Qtemp = Qstl / 10.0 - Qont;
stlont = round(Qtemp); %
stlont = stlont * 10.0; %
6.2 Appendix 2: Summary of UW plan parameters, optimization decision variables and allowable ranges.

Table A2-1 – Details of the decision variables and associated ranges of UW plan for one season

<table>
<thead>
<tr>
<th>No.</th>
<th>Parameter Name</th>
<th>Range</th>
<th>Auxiliary Decision Variable Name</th>
<th>Auxiliary Decision Variable Range</th>
<th>Parameter and auxiliary decision variable relationship</th>
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<td>$h$</td>
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<td>$x_h$</td>
<td>0.5-1.5</td>
<td>$h = x_h * g$</td>
</tr>
<tr>
<td>39</td>
<td>$\text{Baseline Flow}$</td>
<td>0-2</td>
<td>-</td>
<td>-</td>
<td></td>
</tr>
</tbody>
</table>

Table A2-1 presents the decision variables of the optimization problem which determines the UW plan parameter values. As can be seen, there are 39 decision variables for a season to be
determined through optimization; as two seasons were considered in UW plan, the total number of decision variables is 78. According to Table A2-1, some plan parameters are directly optimized and are thus also decision variables with ranges (i.e., parameters $b$, $e$, $f$, $g$, $Alpha$ and $Baseline$ $flow$), while the remaining plan parameters are not decision variables and are instead computed directly from auxiliary decision variables that are optimized. For example, $x_a$ is an auxiliary decision variable for plan parameter $a$ in optimization and $x_a$ can vary in the range of [2-10]. Parameter $a$ is calculated as a function of the parameter $b$ value (a decision variable) and the decision variable $x_a$ via $a = x_a * b$. The auxiliary decision variable approach is utilized to respect the parameter relationships described in Sections 2.4.1 and 2.5.4 such that any set of decision variables generated by the optimizer is always converted to a feasible set of plan parameters. DDS and PADDs, like many global optimization algorithms, are only capable of respecting decision variable bound constraints.
Appendix 3 – NBS scenarios for multi-lake rule curve development

Selected and described by David Fay of Environment Canada (EC) from the 50,000 year stochastic NBS set produced by Fagherazzi et al., (2005) for the LOSL Study. In general, the scenarios were selected to be the most extreme periods of 70 or 80 years of the entire sequence based on certain characteristics.

Selection was based on EC simulated Plan 77A levels (from 2009 version of the 77A CGLRRM). Series are 70-80 years long and each one starts at what are close to initial lake level conditions similar to those used in the base case (e.g., 1900). Selected series (referenced to the years from 1 to 50000 in the stochastic NBS set produced by Fagherazzi et al. (2005) are described as follows:

Combined Superior, MH and Erie levels containing max and min annual average

1. Low: years 35385 to 35454
2. High: years 5703 to 5772

Combined Superior, MH and Erie levels containing max min 30 year moving average

3. Low: years 27001 to 27070
4. High: years 16016 to 16085

Large differences in Sup and MH levels

5. Sup higher: years 25225 to 25304
6. MH higher: years 11668 to 11747

Sets from LOSL study (Reduced to 70 years)

7. Long drought on ONT: 15185-15254

Random set

8. Random selection 14324-14393
6.4 Appendix 4 – Natural (current) connecting channel flow equations.
Open water lake-to-lake equations for natural connecting channel flows are as follows:

Natural instantaneous flow in St. Clair River (m$^3$/s):

\[ Q_{OSC_R} = 186.90 \left( 0.5 \, Z_{MH} + 0.5 \, Z_{SC} - 167.0 \right)^{1.51} \left( Z_{MH} - Z_{SC} \right)^{0.36} - \text{Retardation}_{SCR} \]

Natural instantaneous flow in Niagara River (m$^3$/s):

\[ Q_{ONIR} = 558.3 \left( Z_{ER} - 169.86 \right)^{1.6} - \text{Retardation}_{Nia} \]

$Z_{MH}$, $Z_{SC}$ and $Z_{ER}$ are the average lake levels (m) for lakes Michigan-Huron, St. Clair and Erie while the empirical Retardation factors are reductions in flow due to the presence of ice and weeds.
6.5 Appendix 5 – Rule Curve Parameter Values

Complete multi-lake rule curve parameters for the $6 billion 4-point UW plan

The three tables below detail the exact multi-lake rule curve and can thus be used to simulate the system operation for any NBS scenario. The rule curve equations presented in Section 2.4.1 use these parameters and generate a unitless target release value scaled in the range of zero and one. Transforming this unitless target release value to the original scale in m$^3$/s as outlined in Section 2.5.4 would result in the target release.

Table A5-1 Rule Curve Parameters for the $6 billion UW plan

| Rule Curve Parameters | Lake Superior | | | Lake MH | | | Lake Erie | | | Lake Ontario | | |
|-----------------------|--------------|-----------------|-----------------|--------------|-----------------|-----------------|--------------|-----------------|-----------------|--------------|-----------------|-----------------|--------------|
|                       | Winter       | Summer         | Winter         | Summer       | Winter         | Summer         | Winter       | Summer         | Winter         | Summer       | Winter         | Summer         | Winter       | Summer       |
| $a$                   | 7.3183       | 9.0117         | 0.4850        | 0.6744       | 5.9951        | 5.7035         | 10.8874     | 3.6936         |                |              |                |                |              |              |
| $b$                   | 1.0509       | 0.9740         | 0.2425        | 0.3372       | 0.6974        | 0.5704         | 1.2140      | 1.0377         |                |              |                |                |              |              |
| $c$                   | 1.4643       | 0.6702         | 0.2534        | 0.1905       | 0.9250        | 0.4320         | 1.8210      | 1.2024         |                |              |                |                |              |              |
| $d$                   | 13.6507      | 2.0083         | 0.5068        | 0.8927       | 4.6324        | 4.0392         | 16.2614     | 9.2482         |                |              |                |                |              |              |
| $e$                   | 0.4125       | 0.2000         | 0.9000        | 0.9318       | 0.4070        | 0.2431         | 0.2000      | 0.2000         |                |              |                |                |              |              |
| $f$                   | 0.7274       | 0.9771         | 1.0000        | 0.9953       | 0.9402        | 0.8164         | 0.2000      | 0.3572         |                |              |                |                |              |              |
| $g$                   | -1.4826      | -0.7766        | -0.0737       | -0.1795      | -0.3931       | -0.5652        | -1.2954     | 0              |                |              |                |                |              |              |
| $h$                   | -1.1966      | -0.5451        | -0.0783       | -0.2536      | -0.5896       | -0.4747        | -1.3070     | 0              |                |              |                |                |              |              |
| $\alpha$              | 0.4073       | 0.5665         | 0.9840        | 0.2688       | 0.0067        | 0              | N/A         | N/A            |                |              |                |                |              |              |
| Baseline flow         | 0.7680       | 0.8100         | 0.4082        | 0.4100       | 0.6271        | 0.6043         | 0.2262      | 0.4899         |                |              |                |                |              |              |
Table A5-2 Monthly averaged lake levels (m) over historical period used for rule curve development

<table>
<thead>
<tr>
<th>Lake</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Superior</td>
<td>183.33</td>
<td>183.27</td>
<td>183.24</td>
<td>183.26</td>
<td>183.37</td>
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<td>183.54</td>
<td>183.54</td>
<td>183.51</td>
<td>183.47</td>
<td>183.41</td>
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</tr>
<tr>
<td>MH</td>
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<td>176.29</td>
<td>176.31</td>
<td>176.4</td>
<td>176.49</td>
<td>176.56</td>
<td>176.59</td>
<td>176.57</td>
<td>176.52</td>
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<td>176.34</td>
</tr>
<tr>
<td>St. Clair</td>
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<td>174.9</td>
<td>175.04</td>
<td>175.12</td>
<td>175.17</td>
<td>175.19</td>
<td>175.15</td>
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</tr>
<tr>
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<td>74.68</td>
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<td>75.02</td>
<td>75.05</td>
<td>74.99</td>
<td>74.88</td>
<td>74.74</td>
<td>74.61</td>
<td>74.54</td>
<td>74.53</td>
</tr>
</tbody>
</table>

Table A5-3 Monthly averaged lake inflows (m$^3$/s) simulated for base case over historical period used for rule curve development

<table>
<thead>
<tr>
<th>Lake</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>MH</td>
<td>1921</td>
<td>1895</td>
<td>1875</td>
<td>1944</td>
<td>2125</td>
<td>2199</td>
<td>2282</td>
<td>2359</td>
<td>2326</td>
<td>2243</td>
<td>2207</td>
<td>2029</td>
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<tr>
<td>St. Clair</td>
<td>4589</td>
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<td>4897</td>
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<td>5439</td>
<td>5499</td>
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<td>5457</td>
<td>5406</td>
<td>5353</td>
<td>5188</td>
</tr>
<tr>
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<td>5637</td>
<td>5817</td>
<td>6078</td>
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<td>6306</td>
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<td>6095</td>
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</tbody>
</table>

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