Formulation and Evaluation of New Control Structures in the Great Lakes System

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By:
Dr. Bryan Tolson¹, Saman Razavi² and Masoud Asadzadeh²

1. Assistant Professor
2. PhD Candidates

Department of Civil and Environmental Engineering
University of Waterloo
btolson@uwaterloo.ca

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1 Introduction

1.1 Background

Great Lakes – St. Lawrence water levels are critically important to the Canadian and US economies. Extreme and persistent high or low water levels can have negative impacts on the ecosystem and result in millions of dollars of losses to a variety of stakeholder groups such as hydropower producers, the shipping industry, shoreline property owners and recreational boaters. Although the system is currently managed by control structures at two locations (Lake Superior and Lake Ontario outflows are controlled), there is concern that future extreme climates will generate water supply sequences to the system that will negatively impact the system performance and substantially increase the frequency and persistence of extreme water levels (see for example Angel and Kunkel (2010)).

At the direction of the International Joint Commission (IJC), the International Great Lakes Study (IUGLS) Board is investigating possible changes to the present method of regulating the outflows of Lake Superior to better meet the contemporary needs of the interests and also taking into consideration the potential effects of climate change. The Board, as part of their first Phase completed in December 2009, examined physical processes and possible ongoing changes in the St. Clair River and their impacts on levels of Lake Michigan-Huron. In that effort, the Study Board recognized the importance of climate change, noting the world-wide consensus among scientists that climate change, driven by increasing concentrations of greenhouse gases in the atmosphere, is occurring and will continue, though its effects will differ from one region to another. In their first report to the IJC, the Study Board recommended “…that the need for mitigating measures in the St. Clair River be examined as part of the comprehensive assessment of the future effects of climate change on water supplies in the Upper Great Lakes basin in Report 2 of the Study, on Lake Superior regulation” (International Upper Great Lakes Study Board, 2009).

Recognizing that the water levels and outflows of the Great Lakes are constantly changing in response to the hydroclimatic, hydraulic and meteorological conditions of the basin, the Study Board realized that it would be difficult to design a regulation plan now that would be optimal for all future conditions including climate change. Moreover, it became apparent early in the Study process that Superior regulation alone could do little to reduce the risk posed by extreme water levels downstream of Lake Superior.

With climate change as the driving force, and recognizing the significance of future uncertainties and the attendant inability to craft an Upper Great Lakes regulation plan that could reduce risks from extreme water levels, the Board embraced the notion of Adaptive Management as a means to cope with- and respond to- the inherent uncertainties associated with the system as more is learned over time. Recognizing this was stretching outside the original Plan of Study, which
included studying climate change as it relates to the existing regulation structure at Sault St. Marie, but did not provide generally for considering climate change implications or actions beyond Superior regulation, the Board raised this with the IJC at an October 2009 meeting. The IJC responded in a letter to the Board in April 2010, after consulting with governments, giving their approval for the Study to conduct the examination of climate change impacts on water levels, and indicated it should “include consideration of a full range of options available to all potentially affected sectors across the Great Lakes – St. Lawrence River system at an exploratory level.” Furthermore, guidance from the two governments indicated this exploratory analysis was to include both structural and non-structural measures, and be conducted within the existing Study budget and timeframe.

1.2 Purpose and Scope

As a result of the IUGLS Study Board’s phase 1 recommendations to consider mitigation in the St. Clair River in light of climate change, and the additional guidance in April 2010 from the IJC to examine options to address climate change at an exploratory level, a multi-lake regulation task was added to the Study as part of the Adaptive Management Group work plan. The purpose of this effort would be to provide an exploratory conceptual analysis of how new control structures in the system could be used to try to minimize the risks posed by extreme water levels outside of the historic range. This required a new plan formulation effort to develop multi-lake regulation operating rules, requiring close coordination with the IUGLS Lake Superior Plan Formulation and Evaluation Group. The plan formulation for multi-lake regulation was to consider new hypothetical structures in the St. Clair and/or Niagara rivers that, in combination with the St. Marys River and St. Lawrence control structures (or some revised structures), and possible further mitigating structures downstream of Montreal, would provide a variety of ways to manage levels throughout the Great Lakes that meet a set of hydrologic objectives aimed at minimizing potential risk of extreme water levels. The effect on Lake Ontario and the St. Lawrence River levels would also be considered, or could be specified as constraints. The new alternatives were to be run with a number of water supply scenarios from a suite of stochastic and climate change water supply scenarios with an eye to at least determining how much a multi-lake regulation plan could mitigate extreme levels throughout the system, and perhaps for other objectives, such as moderating levels so they were more like historic levels.

Given that the Study did not want to formulate a multi-lake regulation plan that simply passed damages downstream, the ability to assess the implications for Lake Ontario and the St. Lawrence River was considered critical to the analysis. This however, would limit the number of water supply scenarios available for assessment, as only the water supply scenarios from the Lake Ontario-St. Lawrence River Study (LOSLR) (International Lake Ontario-St. Lawrence River Study Board, 2006) could be utilized, since these are the only water supply scenarios available to the IUGLS for the entire Great Lakes basin that include the Ottawa River flow. This was not considered a major concern, as these scenarios provide numerous extreme conditions to test. Again, as noted in the IJC letter, this would all be done at an exploratory level and would not include detailed benefit/cost assessments. However, the evaluation would make use, to the
extent practical, of existing evaluation models and performance indicators developed for the Lake Superior Plan Formulation and Evaluation component of the IUGLS study. Some preliminary effort would be taken to examine past literature to determine the types of structures and excavation that have been proposed in the past along with any costs estimates updated to current dollar values. No new effort to design structures would be considered. This report details the methods and results of the IJC requested exploratory multi-lake regulation study.

1.3 Reservoir Optimization

Although there are many previous studies on Great Lakes management and multi-lake regulation, most of them were motivated by the occurrence of high water levels while this study was motivated by recent low water levels that occurred over the last ten years. Perhaps the most extensive previous study of multi-lake regulation on the Great Lakes was the IJC Levels Reference Study (Levels Reference Study Board, 1993).

As part of the mandate of the Levels Reference Study, several different initiatives were studied and reported by Task Group 1 of the Working Committee 3 (Task Group 1, Working Committee 3, 1993). The Task Group commissioned an optimization study in order to develop and test a range of system-wide regulation options that included two (lakes Superior and Ontario), three (lakes Superior, Erie and Ontario) and five (Superior, Michigan-Huron, Erie and Ontario) lake regulation. The optimization method was based on the flow network algorithm with a perfect knowledge of net basin supplies for the ninety-year period considered. The objective was based on reducing the cost for the overall system using constraints defined by the six stakeholder interests that were integrated into a weighted single interest satisfaction curve. For example, the optimized results showed a compression of water levels on all lakes (Lake Superior about 9 cm, Lake Michigan-Huron about 52 cm, Lake Erie about 81 cm and Lake Ontario by about 57 cm). The range of water levels, however, at Montreal showed a small increase of 13 cm. The excursions of water levels above and below the critical thresholds, defined by the Citizen Advisory Committee, were also reduced in all lakes while showing an increase at Montreal.

The Levels Reference Study concluded that additional control structures were not economically justified and thus the Study Board did not pursue recommending the construction of any physical works. In fact, the Study Board recommended physical works not be looked at any further. Fletcher and Ponnambalam (1998) followed up the work done in the Levels Reference Study and present an improved methodology and then demonstrate it using a simplified case study on the Great Lakes. Another important study on the Great Lakes system is the Lake Ontario – St. Lawrence study (International Lake Ontario-St. Lawrence River Study Board, 2006) that focuses on improving the current Lake Ontario regulation plan.

A variety of other studies have also optimized the operation of the existing control points on the Great Lakes. Seifi and Hipel (2001) applied two stage stochastic linear programming with recourse to the Great Lakes System (Lake Superior through Lake Ontario considered). Seifi and Hipel (2001) formulated an optimization model that yields an optimal annual set of monthly
release targets for the existing control points (Lake Superior and Lake Ontario) across multiple water supply scenarios. Their objective was to minimize the sum of absolute deviations from regulatory storage and release targets and the nonlinear routing equations for connecting channel flows were approximated using a first-order Taylor series approximation.

There is a rich history of reservoir optimization modelling and it is beyond the scope of this study to review it. Labadie (2004) provides a thorough critical review of reservoir optimization modeling and the possible range of approaches. Very recently water resource system and reservoir optimization studies have started to tackle the question of optimal reservoir management under future climate change projections. Examples include the work of Vicuna et al. (2010), Lee et al. (2009) and Lee et al. (In press).

The specific goal of this multi-lake regulation study is to determine to what extent the base case management system levels can be improved when additional control structures are added on the Upper Great Lakes. In particular, this study focuses on developing multi-lake regulation strategies to mitigate extreme climates, or more specifically, extreme water supply scenarios. The additional control structures and channel changes considered are in the St. Clair River to control the outflow of Lake Michigan-Huron (MH) and in the Niagara River to control the outflow of Lake Erie.

Our approach to multi-lake optimization was to develop a set of multi-lake rule curves defining the target releases for each control point. We selected to first formulate the mathematical structure of these rule curves and then optimize the rule curve coefficients directly via global optimization methods instead of the more traditional approach of formulating an optimization model to optimize releases directly and then fitting releases via some sort of multivariable regression analysis. The main benefit of our approach is that it provides a maximum amount of flexibility in terms of being able to directly simulate for any set of control points the non-linear routing and water levels of the Great Lakes – St. Lawrence system (instead of approximating these values) and in terms of the objective functions utilized. The other obvious benefit is that the optimization result is a release policy that can be simulated for any other NBS sequence. As noted by Labadie (2004), inferring release rules from an optimal sequence of releases by regression analysis can fail and the alternative of developing release rules based on optimal releases from other methods can require extensive trial and error experimentation and can have little general applicability. A variety of previous studies have solved the reservoir optimization problem with an approach consistent with the general approach to optimize rule curves utilized in this study. Some example studies directly optimizing rule curve coefficients or rule curve structures with single or multi-objective heuristic global optimization algorithms include Oliveira and Loucks (1997), Chen et al. (2007), Valeriano et al. (2010), Afshar et al. (2011) and Li et al. (2010).
1.4 Outline of Report

The remainder of this report is structured as follows. Section 2 describes the details of the simulation-optimization methodology utilized to develop optimized multi-lake rule curves. This includes a description of the three different regulation and hydrologic routing simulation models invoked during the optimization process, development of the primary optimization objective, as well as the secondary objective and the constraints in the optimization formulation, and highlights how multiple water supply sequences were incorporated into the optimization formulation. Results are described in Section 3 and all of these compare various UW plans using *four* control points to the base case performance of the current regulation strategy (two control points). Section 3.1 details the single objective optimization results for the primary objective (keeping lake levels within historical extremes) and compares the University of Waterloo (UW) plans with and without the lower St. Lawrence River locations in the formulation. Section 3.2 evaluates performance of one of the optimized UW plans against the base case plan for a 50,000 year water supply sequence. Note that the UW plan was only optimized to eight approximately 70 year water supply sequences selected from this 50,000 year sequence and thus results in Section 3.2 show the robustness of the UW plan for water supply sequences the plan was not optimized for. Section 3.3 describes the multi-objective optimization results or tradeoff between the primary objective (keeping lake levels within historical extremes) and the secondary objective (structural + dredging costs associated with optimized releases). Section 4 lists the ongoing work for this project and then conclusions are provided in Section 5.
2 Simulation-Optimization Methodology

The first step in this multi-reservoir optimization study was to define the simulation models used to calculate lake levels and lake outflows across the system. These simulation models varied depending on what multi-lake regulation policy is to be simulated. A total of three different simulation models (each of which is a combination of a variety of simulation codes) were developed in this study and these are described in detail in Sections 2.1, 2.2 and 2.3. These three simulation models are then utilized in a variety of ways to assess system performance of multiple multi-lake regulation plans as defined by multi-lake rule curves that are described in Section 2.4. This assessment can be conducted within or independent of the optimization models formulated to optimize the management objectives and constraints described in Sections 2.5.1 through 2.5.5.

2.1 Base case regulation and hourly hydrologic routing model

The Great Lakes – St. Lawrence system with the current regulation strategies is called the base case. From Lake Superior down to the Niagara River, the base case is simulated with the Coordinated Great Lakes Routing and Regulation Model (CGLRRM) that was originally developed by the National Oceanographic and Atmospheric Administration’s (NOAA’s) Great Lakes Environmental Research Laboratory (GLERL), in conjunction with the US Army Corps of Engineers (USACE) and Environment Canada (EC). The origin of CGLRRM was through work by Quinn (1978) and refined later by Clites and Lee (1998). Today, CGLRRM is sanctioned by all relevant agencies from Canada and the US that make up the Coordinating Committee on Great Lakes Basic Hydraulic and Hydrologic Data. Also Plan 58DD is used to simulate Lake Ontario regulation in the base case. The base case simulation is further extended to the St. Lawrence system with various empirical equations. This simulation approach is consistent with the simulation approach used in the Lake Ontario St. Lawrence River (LOSLR) study (International Lake Ontario-St. Lawrence River Study Board, 2006).

Simulation of Lake Superior

CGLRRM uses Plan 77A to regulate Lake Superior and calculates the outflow and surface elevation of the lake at the end of each time step.

Simulation of Mid-Lakes (Lake MH down to Niagara River)

CGLRRM uses a hydrologic routing model of the Great Lakes and is comprised of continuity equations for each lake, solved using a second-order finite-difference technique (Quinn, 1978) with a numerical time step of one hour. It uses as inputs time series of net basin supply (NBS) and any diversions, as well as initial outflows and lake levels, for each lake. The model calculates discharge from each lake from an open-water lake-to-lake stage-fall-discharge equation with regression-based coefficients. In the winter, flows in the connecting channels are naturally reduced by the presence of ice. CGLRRM does not explicitly simulate the ice retardation process and instead this phenomenon must be described by a time series of ice retardation rates (e.g., reduction in connecting channel flow in m$^3$/s or cms) for each channel. Caldwell (2008) provides a detailed description of the ice retardation approach in CGLRRM and
assesses the long term impacts of apparent roughness factors, which include ice retardation on lake levels.

CGLRRM simulates the surface elevation of Lakes Michigan-Huron (considered to be a single lake, MH, for hydraulic purposes), St. Clair and Erie and uses as input the estimated flows of the St. Marys River. The Chicago and Welland Canal diversions are also inputs to the model (coordinated monthly averages). Most inputs for CGLRRM in all analyses in this report are specified on a monthly time step for Lake Superior outflow (St. Marys River coordinated monthly averages flow), residual NBS of Lake MH and St. Clair, ice and weed retardation of St. Clair and Detroit Rivers. CGLRRM inputs for residual NBS of Lake Erie and ice and weed retardation of Niagara River were specified at a quarter monthly time step. Also the Chicago diversion is constant (91 cms) and the Welland Canal diversion is monthly constant (220, 211, 222, 247, 244, 246, 248, 239, 244, 243, 243, 237 cms) for the whole simulation period. Outputs of CGLRRM include monthly mean lake levels, beginning-of-month levels, and monthly flow rates for the connecting channels.

Simulation of Lake Ontario

In the base case, Lake Ontario levels are simulated by Lake Ontario regulation plan (Plan 58DD) that controls outflows from the lake and is simulated based on a compiled program provided by Environment Canada.

St. Lawrence River System Simulation

Simulation of the St. Lawrence River system is consistent with the simulation approach used in the LOSLR study (International Lake Ontario-St. Lawrence River Study Board, 2006). This part of the simulation uses quarter monthly inputs to calculate quarter monthly surface elevations along the St. Lawrence River-reservoir system. The hydraulic behavior of the St. Lawrence River system is extremely complex and thus the empirical approach used here represents a substantial simplification of the system. As such, simulation of this part of the system is less accurate than that of Great Lakes system.

St. Lawrence system simulation has two parts. In the first part, surface elevations on various locations along the St. Lawrence system down to Saunders Headwater are calculated based on empirical equations. Among these locations, Iroquois Headwater Level and Saunders Headwater Level are required for the optimization. The following empirical equations show how to estimate St. Lawrence River elevations (in m) at Iroquois Headwater (IHW) and the Saunders Headwater (SHW) evaluation points:

\[ Kingston = \text{Lake Ontario mean level} - 0.03 \text{ m} \quad (1) \]
\[ IHW = Kingston - N_{IHW} \left[ Q / (23.6495 \times (Kingston - 62.40)^{2.2886}) \right]^{(1/0.4158)} \quad (2) \]
\[ SHW = Kingston - N_{SHW} \left[ Q / (21.603 \times (Kingston - 62.40)^{2.2586}) \right]^{(1/0.3749)} \quad (3) \]
Where \( N \) is the apparent roughness factors that vary quarter-monthly due to ice, weeds, winds etc. and is given for the 50,000 years of stochastic data and \( Q \) is the Lake Ontario outflow in cms which is calculated by the rule curve of Lake Ontario. It should be noted that when SHW level are computed greater than 73.87 m from the above equations, the SHW level is set to a value of 73.78 m. This rule is based on how the system is operated currently to avoid high levels at SHW.

Level of Lac St. Louis at Pointe Claire is also required in the objective function calculation. This level is calculated by the following equation:

\[
H_{PCL} = 16.57 + (F_{STL} \times Q_{STL} / 604.0)^{0.580}
\] (4)

Where \( F_{STL} \) is the dimensionless ice factor for Lac St. Louis and \( Q_{STL} \) is outflow from Lac St. Louis in cms. Time series of \( F_{PCL} \) and \( Q_{STL} \) are available for 50,000 years of stochastic data.

In the second part, the St. Lawrence system downstream of Lac St. Louis is simulated based on empirical equations used in the LOSL study. From this part of the simulation, only Montreal harbour level (at Jetty 1) is required in the optimization. The following empirical equation from LOSL study is used to calculate this level.

\[
H_{MTL} = [K_{MTL}(0.001757Q_{STL} + 0.000684Q_{DPMI} + 0.001161Q_{STFRN} + 0.000483Q_{STMAU})]^{0.6589} + 0.9392T
\] (5)

where \( K_{MTL} \) and \( T \) are dimensionless ice-weed factors and tidal signal for Montreal harbour respectively. \( Q_{STL} \) is Lake St. Louis outflow, \( Q_{DPMI} \) is outflow of the Des Prairies plus the Mille Iles, \( Q_{STFRN} \) is outflow of the Saint-Francois River at Hemming Falls, \( Q_{STMAU} \) is the outflow of the St. Maurice River at La Gabelle all in cms. Time series of 50,000 years of stochastic data includes all these required parameters and flows to calculate the level at Montreal harbour.

### 2.2 Two-point Multi-lake regulation and daily hydrologic routing model

As a part of this study, the current regulation of Lake Superior and Lake Ontario is replaced with new rule curves for these locations. In other words, the same set of existing structures is assumed to exist but the only change to the system is to optimize new operating rules at these locations. The two-point multi-lake regulation and daily hydrologic routing model simulates levels under such a regulation framework. Simulation of the system with two control points (one on Lake Superior and one on Lake Ontario) is coded in MATLAB based on the hydrologic routing component of CGLRRM and a mass balance for Lakes Superior and Ontario. This section describes the simulation of this system and its differences to the base case described in Section 2.1. Similar to the base case, this simulation is also extended to the St. Lawrence system with the same empirical equations.

**Simulation of Lake Superior**

Outflow of Lake Superior respects the regulation rules coded in CGLRRM that split the lake outflow between the side channels (for hydropower) and gates (flow through the rapids).
Therefore the outflow of Lake Superior assigned by the rule curve (or any release policy) is always adjusted to respect the limits associated with the side channel and gate split. Knowing the initial surface elevation of Lake Superior, the actual release amount and the NBS for the current time step, the mass balance equation is applied to calculate the surface elevation of Lake Superior at the end of each time interval (daily time step in our simulation).

**Simulation of Mid-Lakes (Lake MH down to Niagara River)**

Simulation of this part of the system is not affected by the modifications in the regulation of Lake Superior and Ontario compared to the base case. Therefore, it calculates discharge from Lake MH, St. Clair and Erie from open-water lake-to-lake stage-fall-discharge equations (the same equations as the base case simulation model). This results in a system of three linear equations with three unknowns that are surface elevation changes of Lakes MH, St. Clair and Erie for each numerical time step. This system of equations is solved iteratively in the base case (by CGLRRM). Note that in our two-point simulation model, the matrix solution approach is utilized since it is much more computationally efficient. Moreover, in the base case, the numerical time step of one hour is used for the most accurate results; while here, a numerical time step of one day is used to make the simulation more computationally efficient. This efficiency gain comes at very little change in solution quality (e.g., average monthly lake levels changing by only a few cm at worst).

Just like the base case simulation model description in Section 2.1, the two-point simulation model uses as inputs an NBS data set, diversions, as well as initial outflows and lake levels for each lake, and a time series of ice retardation rates (e.g., reduction in connecting channel flow in cms) for the St. Clair, Detroit and Niagara Rivers.

The Chicago and Welland Canal diversions are also inputs to the model (coordinated monthly averages). Most inputs for simulation of Great Lakes system in all analyses in this report are specified on a monthly time step for residual NBS of Lakes Superior, MH and St. Clair, ice and weed retardation of SCR and Detroit River. Inputs for residual NBS of Lake Erie and ice and weed retardation of Niagara River were specified at a quarter monthly time step. Also the Chicago diversion is constant (91 cms) and the Welland Canal diversion is monthly constant (220, 211, 222, 247, 244, 246, 248, 239, 244, 243, 243, 237 cms) for the whole simulation period. Outputs of the simulation model include monthly mean lake levels, beginning-of-month levels, and monthly flow rates for the connecting channels.

**Simulation of Lake Ontario**

There are some physical/structural limits that Lake Ontario outflow must respect. These limits are computed based on the Lake Ontario regulation plan (Plan 58DD) provided by Environment Canada. The release value (target release) obtained from the Lake Ontario rule curve is modified (actual release) to respect the limits of Lake Ontario releases. The actual releases from Lake Ontario are governed by the target release as specified by a rule curve/release policy and the Lake Ontario release constraints. Quarter-monthly releases from Lake Ontario must satisfy a complex suite of physical or structural constraint limits explained in Fay (Annex 3, International
Lake Ontario St. Lawrence River Study, 2005). For example, these constraints include respecting maximum week to week flow changes and limiting flows during ice formation periods. Knowing the initial surface elevation of Lake Ontario, the actual release amount from Lake Ontario and the NBS for the current time step, the mass balance equation is applied to calculate the surface elevation of Lake Ontario at the end of each time numerical time step (daily in our simulation).

*St. Lawrence System Simulation*

Simulation of this part of the system is not affected by the modified regulation of Lakes Superior and Ontario; therefore, this part of the system is simulated exactly the same as in the base case.

### 2.3 Four-point Multi-lake regulation daily hydrologic routing model

In order to have more flexibility for the multi-lake regulation, two new control points are to be considered at the outlet of Lakes MH and the outlet of Lake Erie. Hence, the Hydrologic routing part of the simulation model should be modified to only simulate Lake St. Clair and Lake Erie surface elevations and St. Clair River flow. Here, surface elevation of Lakes Superior and MH are calculated by mass balance. This section explains the required changes to the simulation model of the system with four control points with more details.

*Simulation of Lake Michigan-Huron*

The rule curve of Lake MH determines its outflow. Therefore, all inflows and outflows of the lake are known and mass balance is applied to calculate the end of interval Lake MH surface elevation.

*Modified Hydrologic Routing Model for the System with Four Control points*

The rule curve of Lake MH and Erie define the outflows of these two lakes and the surface elevation of Lake MH. Therefore, the Hydrologic routing model should be modified to calculate only surface elevations of Lake St. Clair and Lake Erie and the Detroit River flow. Hence the system of linear equations with two unknowns must be solved here to calculate the changes in the surface elevation of Lakes St. Clair and Erie and lake levels at the end of a numerical time step. The system of two linear equations is reported in the appendix 1 of the report.

*S some Concerns in Simulation of the System with Four Control Points*

When the rule curve defines the outflow of Lake MH, the open-water lake-to-lake stage-fall-discharge equation between Lake MH and St. Clair does not necessarily result in the St. Clair River flow which is determined by the rule curve. This suggests that the conveyance capacity of St. Clair River must be changed. This issue will be addressed in Section 2.5.5 where an estimate of the cost of structural channel changes to the system is estimated. Another concern is that, in some extreme situations, Lake MH outflow is so high that it decreases the surface elevation of Lake MH and increases the surface elevation of Lake St. Clair so that surface elevation of Lake St. Clair becomes higher than that of Lake MH. The simulation model in this study is not able to handle this issue and will fail. One way to avoid this situation is to reduce the outflow of Lake MH in the corresponding time step. In this study, the outflow of Lake MH is reduced to zero in
the corresponding numerical time step (day). Similar situations may occur between Lake St. Clair and Lake Erie. One way to resolve this issue is to reduce the flow of corresponding connecting channel to zero. However, if the connecting channel flow in such situations is already small, this strategy may not resolve this issue.

2.4 Multi-Lake Rule Curve Development

In this study, various regulation plans, which are all referred to as “UW plan” hereafter, consisting of a set of rule curves for operation of the entire Great Lakes-St. Lawrence System are developed. The UW plans reported on here are designed to regulate the two existing control structures downstream of Lake Superior and Lake Ontario (Moses Saunders Dam) as well as two hypothetical (potential) control structures downstream of Lake Michigan-Huron and Lake Erie. Later work will develop plans for only two or three control points. All physical/hydraulic constraints existing in Lake Superior and Lake Ontario control structures are considered and respected in the UW plans.

2.4.1 Form of Rule Curves

Four rule curves to make release decisions corresponding to the four control points (Superior, Michigan-Huron, Erie, and Ontario) were generated. Release decisions are made starting at Superior and then sequentially moving downstream. As such, for example, the Superior planned release at time \( t \) is known when the release for time \( t \) from Michigan-Huron is being determined. The regulation time step is quarter monthly (a new release is computed after each quarter month) at all control points (compared to the daily numerical routing time step) except for Lake Superior where the regulation time step is monthly to be consistent with the current regulation time step in Plan77A. The rule curve target releases are defined by up to three additive components quantifying the storage condition upstream and downstream of a control point as well as a baseline flow term. The exact form of the rule curve at control point \( i \) \((i=1, 2, 3, \text{and } 4)\) is given as follows:

\[
\text{Target Release}(i, t) = \text{Component 1}(i, t) + \text{Component 2}(i, t) + \text{Component 3}(i, t) + \text{Baseline Flow}(i, t) \quad (6)
\]

where component 1 is a function of the upstream storage condition of control point \( i \), component 2 is a function of the storage condition downstream of control point \( i \) down to the next control point, \( i+1 \), and component 3 is a function of the storage condition between the next two control points, \( i+1 \) and \( i+2 \). For convenience, all these components and the target release were assumed to be in the range of zero to one. The 0-1 unit-less target release calculated from equation (1) is transformed into units of \( \text{m}^3/\text{s} \) as described in Section 2.5.4 for each lake. Components 1, 2, and 3 are explained in the following subsections.

Component 1:

Component 1 is a piecewise linear function of the upstream storage condition with three break points as shown in Figure 1. The x-axis represents the water storage condition upstream of control point \( i \) \((i=1, \ldots, 4 \text{ for Superior, MH, Erie, and Ontario})\) which can be calculated as:
UpstreamStorage(1) = A_{Sup} (Z_{Sup} - \text{avg}Z_{Sup}) \quad (7)

UpstreamStorage(2) = (RV_{Sup} - \text{avg}RV_{Sup}) + A_{MH} (Z_{MH} - \text{avg}Z_{MH}) \quad (8)

UpstreamStorage(3) = (RV_{MH} - \text{avg}RV_{MH}) + A_{SC} (Z_{SC} - \text{avg}Z_{SC}) + A_{ER} (Z_{Er} - \text{avg}Z_{Er}) \quad (9)

UpstreamStorage(4) = (RV_{Er} - \text{avg}RV_{Er}) + A_{ON} (Z_{ON} - \text{avg}Z_{ON}) \quad (10)

where A_{Sup}, A_{MH}, A_{ER}, and A_{ON} are the lake area of Superior, Michigan-Huron, Erie, and Ontario, respectively. Z_{Sup}, Z_{MH}, Z_{SC}, Z_{ER}, and Z_{ON} are the lake levels at the beginning of the time interval. \text{avg}Z_{Sup}, \text{avg}Z_{MH}, \text{avg}Z_{SC}, \text{avg}Z_{Er}, and \text{avg}Z_{ON} are the historical average monthly lake levels. RV_{Sup}, RV_{MH}, and RV_{Er}, are the release volume planned to be released from Superior, Michigan-Huron, and Erie during the next time interval, and \text{avg}RV_{Sup}, \text{avg}RV_{MH}, and \text{avg}RV_{Er} are the historical average monthly flow volume in that interval.

Figure 1- Component 1 representing the contribution of upstream storage condition on regulated release

According to Equations 2-5, upstream storage condition in Lake Superior is only a function of current Superior lake level, but in the other control points, the planned release from the upper control point is also considered in the upstream storage condition confirming that release decisions made upstream sequentially affect the release decisions downstream.

Upstream storage of zero suggests that the control point is in normal condition (i.e., no excess or shortage of water). Any positive value for upstream storage suggest that the control point is at the excess water condition (i.e., water budget available is above than average historical records), and a negative number for upstream condition suggest that there is a shortage of water at the control point.

Component 1 consists of four segments: high shortage, low shortage, low excess, and high excess, resulting from three break points. Component 1 responds differently (different associated slopes in the piecewise linear function) for different upstream storage conditions. As shown in Figure 1, a, b, c, and d are the slope values associated to high excess, low excess, low shortage, and high shortage, respectively.
high shortage cases, respectively. The component 1 slope values are positive following the direct relationship between upstream water storage and release. In other words, the higher the water storage is, the higher the release is. The following assumptions were also considered for determination of these slopes: \( b \) and \( c \) are supposed to have relatively close values; high excess segment is supposed to have a considerably steeper slope than the low excess segment (\( a > b \)), and high shortage segment is supposed to have a considerably steeper slope than the low shortage segment (\( d > c \)). When the upstream storage is in high excess or high shortage cases, it means that the lake level is well above or well below the normal level and it is going (or has gone) beyond the desirable lake level range. As such, the justification of \( a \) and \( d \) having considerably steeper values is to avoid going beyond the desirable range as component 1 has a direct linear contribution in determining the target release (see equation 1).

The center breakpoint is located at upstream storage of zero (normal condition). As shown in Figure 1, the breakpoints specifying the high shortage and high excess conditions are determined through location parameters \( e \) and \( f \). Overall, component 1 has six parameters, including four slope parameters \( a, b, c, \) and \( d \) as well as two location parameters \( e \) and \( f \), to be determined through optimization.

**Component 2:**

Component 2 is a two-segment linear function of the storage condition immediately downstream of a control point down to the next control point as shown in Figure 2. For control point \( i \) (\( i = 1, 2, 3, \) and \( 4 \)), the x-axis is the current water budget available between control point \( i \) and control point \( i + 1 \) (next control point) being calculated through the following equations:

\[
\text{DownstreamStorage}_\text{im}(1) = A_{\text{MH}} (Z_{\text{MH}} - \text{avg}Z_{\text{MH}}) \tag{11}
\]

\[
\text{DownstreamStorage}_\text{im}(2) = A_{\text{SC}} (Z_{\text{SC}} - \text{avg}Z_{\text{SC}}) + A_{\text{ER}} (Z_{\text{Er}} - \text{avg}Z_{\text{Er}}) \tag{12}
\]

\[
\text{DownstreamStorage}_\text{im}(3) = A_{\text{ON}} (Z_{\text{ON}} - \text{avg}Z_{\text{ON}}) \tag{13}
\]

\[
\text{DownstreamStorage}_\text{im}(4) = Z_{\text{Jetty1}} - \text{avg}Z_{\text{Jetty1}} \tag{14}
\]

where \( Z_{\text{Jetty1}} \) is the water level at Jetty1 station and \( \text{avg}Z_{\text{Jetty1}} \) is the historical average monthly levels at Jetty1. Other terms in the above equations were already defined for equations 2-5.

DownstreamStorage_im of zero indicates that the water storage condition in the immediate downstream of a control point is normal; a positive storage value means the water available is greater than normal and a negative value indicates the available water is less than the normal condition. As can be seen in Figure 2, component 2 has two segments (with two different slopes) and the breakpoint is located at the storage of zero. The slope values, \( g \) and \( h \), are negative to reflect the inverse relationship between downstream storage and release. In other words, the less the downstream storage is the more the release is. The component 2 slope values \( g \) and \( h \) are parameters of the rule curve and should be determined through optimization.
Component 3:

Component 3 for control point \( i \) was designed to represent the storage condition downstream of the next control point (i.e., between control points \( i+1 \) and \( i+2 \)). The storage condition between control points \( i+1 \) and \( i+2 \) can be calculated through the following equations:

\[
\text{DownstreamStorage}_{\text{far}}(1) = A_{\text{SC}} (Z_{\text{SC}} - \text{avg}Z_{\text{SC}}) + A_{\text{ER}} (Z_{\text{ER}} - \text{avg}Z_{\text{ER}}) \quad (15)
\]

\[
\text{DownstreamStorage}_{\text{far}}(2) = A_{\text{ON}} (Z_{\text{ON}} - \text{avg}Z_{\text{ON}}) \quad (16)
\]

\[
\text{DownstreamStorage}_{\text{far}}(3) = Z_{\text{Jetty1}} - \text{avg}Z_{\text{Jetty1}} \quad (17)
\]

Note that component 3 is not applicable for control point 4 being Lake Ontario regulation structure. The terms in above equations were already defined for equations 2-9.

Similar to Component 2, Component 3 has two negatively sloped segments with a breakpoint located at the water storage of zero (see Figure 2); however, unlike Component 2, Component 3 is not always active. Component 3 involves an if-then statement to be calculated as:

IF the lakes between control points \( i \) and \( i+1 \) and the lakes between control points \( i+1 \) and \( i+2 \) have the same storage condition (both in shortage or both in excess)  
calculate Component 3 from Component 3 equation (as shown in Figure 2)  
ELSE Component 3 = 0

Figure 2- Component 2 and component 3 representing the contribution of downstream storage condition on regulated release

The logic behind this if-then statement is explained through an example. Suppose that at the beginning of a Superior release decision time interval Lake MH is in excess condition (wet) while the set of Lake St Clair and Lake Erie is in shortage condition (dry). In such case, Component 3 would become zero since any extra water released from Superior for Lake St. Clair and Lake Erie would first exacerbate the excess water condition on Lake MH. Instead, in such a case, the already available excess water on Lake MH can be released for downstream to Lake St Clair and Lake Erie.
Moreover, as defined in Figure 2, Component 3 has only one parameter, $\alpha$, varying between 0 and 1. This parameter is multiplied by the slope parameters of Component 2, $g$ and $h$, to determine the slopes in Component 3 equation. As such, Component 3 is limited to have an equal or smaller impact (per unit change in storage condition) on release decision than Component 2.

2.4.2 Rule Curve Seasonality

Two regulation seasons in a year were considered for all control points in rule curve development as follows:

- For Lake Superior, the Summer Season is May through November, and Winter Season is December through April.
- For Lake MH and Lake Erie, ice-free season is 3rd quarter month of April through 3rd quarter month of December, and ice season is 4th QM of December to 2nd QM of April (suggested by David Fay of Environment Canada, personal communication)
- For Lake Ontario, shipping season is 4th quarter month of March through 3rd quarter month of December, non-shipping season is 4th quarter month of December through 3rd quarter month of March (suggested by D. Fay of Environment Canada, personal communication)

2.5 Optimization Formulation

As presented in Section 2.4.1, for a single season, the number of parameters to be optimized in the rule curve equations with four control points (Lake Superior, MH, Erie, and Ontario) are 10, 10, 10, and 9, respectively. As two seasons were assumed for regulation of each control point, there are 78 rule curve parameters (i.e., decision variables) in total to be optimized.

2.5.1 Primary Objective of Multi-lake Regulation

The ultimate objective of the project is to evenly enhance the status of water levels in the Great Lakes Saint Lawrence System so that all potential benefits resulting from any system improvement should be fairly shared among all and expenses associated with possible severe future climate should be fairly distributed. The water level at nine main components of the system including Lake Superior, Lake Michigan-Huron, Lake St. Clair, Lake Erie, Lake Ontario, Iroquois Headwater, Saunders Headwater, Pointe Claire on Lake St. Louis, and Jetty1 at Montreal were selected as performance evaluation points, each of which is a representative of a group of stakeholders.

The primary objective of the UW plan is to maintain the water level at these nine evaluation points within the range simulated for the historical period (1900-2008) when experiencing any future climate condition. Table 1 reports the extreme levels over the historical period of 1900-2008 as simulated through the current system (i.e., with the current control structures) and current regulation plan (77A+58DD). As maintaining the water level in these specified ranges under a severe future climate is not always possible, the UW plan focuses to improve the system (i.e., reducing the frequency of being beyond extremes) over what is called “Base Case” in this report. Base Case provides a baseline of performance comparison and refers to performance of
the system with only the current two structures (Lake Superior and Lake Ontario) that are operated/regulated under the current regulation plan. Overall, the objective function is constructed to try to identify a plan that improves upon the base case regulation performance (reduced frequency of going beyond extremes) at every evaluation location for every NBS scenario considered.

Table 1. Min and Max monthly extreme simulated average levels at various evaluation locations for the historical period (1900-2008). Extremes based on simulation results provided by Environment Canada (personal communication, Yin Fan).

<table>
<thead>
<tr>
<th>No.</th>
<th>Type</th>
<th>Location</th>
<th>Simulated Period</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Lake Superior</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>183.70</td>
<td>183.63</td>
<td>183.61</td>
<td>183.67</td>
<td>183.72</td>
<td>183.73</td>
<td>183.76</td>
<td>183.78</td>
<td>183.82</td>
<td>183.79</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Michigan-Huron</td>
<td>1950-2008</td>
<td>Min Extreme</td>
<td>182.91</td>
<td>183.04</td>
<td>183.08</td>
<td>183.83</td>
<td>182.94</td>
<td>182.99</td>
<td>183.00</td>
<td>183.03</td>
<td>183.02</td>
<td>183.17</td>
<td>183.05</td>
<td>183.08</td>
</tr>
<tr>
<td>3</td>
<td>Lake St. Clair</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>177.15</td>
<td>177.07</td>
<td>177.08</td>
<td>177.16</td>
<td>177.23</td>
<td>177.30</td>
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<td>177.36</td>
<td>177.39</td>
<td>177.45</td>
<td>177.37</td>
<td>177.35</td>
</tr>
<tr>
<td>4</td>
<td>Lake Erie</td>
<td>1950-2008</td>
<td>Min Extreme</td>
<td>175.30</td>
<td>175.39</td>
<td>175.42</td>
<td>175.46</td>
<td>175.50</td>
<td>175.60</td>
<td>175.63</td>
<td>175.53</td>
<td>175.46</td>
<td>175.41</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Lake St. Clair</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>174.86</td>
<td>174.87</td>
<td>174.87</td>
<td>174.97</td>
<td>175.05</td>
<td>175.02</td>
<td>175.00</td>
<td>174.93</td>
<td>174.80</td>
<td>174.83</td>
<td>174.88</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>Lake Ontario</td>
<td>1950-2008</td>
<td>Min Extreme</td>
<td>173.42</td>
<td>173.31</td>
<td>173.42</td>
<td>173.53</td>
<td>173.63</td>
<td>173.70</td>
<td>173.66</td>
<td>173.63</td>
<td>173.52</td>
<td>173.43</td>
<td>173.42</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>Iron ore Dam IH</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>172.71</td>
<td>172.54</td>
<td>172.54</td>
<td>172.54</td>
<td>172.63</td>
<td>172.62</td>
<td>172.52</td>
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<td></td>
</tr>
<tr>
<td>8</td>
<td>Saunders IH</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>171.78</td>
<td>171.78</td>
<td>171.78</td>
<td>171.72</td>
<td>171.71</td>
<td>171.70</td>
<td>171.69</td>
<td>171.68</td>
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<td>171.66</td>
<td>171.65</td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>JETTY at Montreal</td>
<td>1950-2008</td>
<td>Max Extreme</td>
<td>170.00</td>
<td>170.08</td>
<td>170.08</td>
<td>170.06</td>
<td>170.06</td>
<td>170.05</td>
<td>170.04</td>
<td>170.03</td>
<td>170.02</td>
<td>170.01</td>
<td>170.00</td>
<td></td>
</tr>
</tbody>
</table>

Initially, a single NBS scenario based objective function was developed and tested as a proof of concept. It was then extended to a multi-NBS scenario approach to cover different possible future severe conditions and circumstances. The following sub-sections present the details of both approaches.

2.5.2 Single NBS Scenario Approach

An objective function was designed first to optimize the rule curve parameters for any single NBS scenario. As explained in Section 1.1, the system performance in the base case is the baseline in the objective function. Therefore, \( A \) is a given set of data for a specific NBS scenario as:

\[
A = \{a_j | j = 1, 2, \ldots, n\}
\]

where \( a_j \) is the number of times in base case that level at evaluation point \( j \) (obtained from plan 77A+58DD) violates the historical simulated extremes.

Assuming \( X \) is vector containing the 78 decision variables (i.e., rule curve parameters), \( Y = f(X) \) where \( f \) is the UW plan based simulation, and

\[
Y = \{y_j | j=1, 2, \ldots, n\}
\]

where \( y_j \) is the number of times in UW plan that level at evaluation point \( j \) violates the historical extremes, and \( n \) is the number of evaluation points (i.e., Lake Superior, Lake MH, ...).
The objective function is:

\[
\text{Minimize } \sum_{j=1}^{n} z_j (y_j - a_j)^2 - \sum_{j=1}^{n} (1 - z_j) \sqrt{a_j - y_j} 
\]

\( Z_j = 0 \) if \( y_j < a_j \) (performance in point \( j \) better than baseline)

\( Z_j = 1 \) if \( y_j \geq a_j \) (performance in point \( j \) worse than baseline)

The square and square root functions in the objective function help the optimizer balance the benefits and costs across all evaluation points in a way that:

- The first priority is given to the evaluation points which experience the highest level of inferiority compared to base case
- The second priority is given to the evaluation points which are still inferior compared to base case but the degree of inferiority is reduced
- The third priority which is completely dominated by first and second priorities is given to evaluation points which are slightly better than base case.
- The forth/last priority is given to the evaluation points which are substantially better than base case.

2.5.3 Multi NBS Scenario Approach

The rule curve parameter values optimized through the single NBS scenario formulation may suffer from a lack of generalization. In other words, the rule curve would perform very satisfactorily on the NBS scenario used for optimization and other NBS scenarios with similar characteristics. However, it may perform poorly on NBS scenarios with significantly different behaviors. Thus, a multi NBS scenario formulation was developed to optimize the rule curve parameters over multiple NBS scenarios with significantly different behaviors simultaneously. The resulting rule curves would be expected to be robust and more reliable when facing unpredictable future climate conditions. The multi-scenario formulation is as follows:

\( n \): number of evaluation points (i.e., Lake Superior, Lake MH, …)

\( m \): number of NBS scenarios to optimize a single regulation plan for \( A = \{a_{j,k} | j = 1, 2, …, n \text{ & } k = 1, 2, …, m\} \)

\( a_{j,k} \): base case performance on evaluation point \( j \) in scenario \( k \)

\( Y = \{y_{j,k} | j = 1, 2, …, n \text{ & } k = 1, 2, …, m\} \)

\( y_{j,k} \): UW plan performance on evaluation point \( j \) in scenario \( k \)

The multi-scenario objective function is:

\[
\text{Minimize } \sum_{j=1}^{n} \left\{ z_j \left( \max_k (y_{j,k} - a_{j,k}) \right)^2 - (1 - z_j) \left( \min_k (a_{j,k} - y_{j,k}) \right)^{0.5} \right\} 
\]
As can be seen, this optimization formulation is a minmax/maxmin optimization problem trying to minimize the maximum inferiority and maximize the minimum improvement in a way that:

- If the new plan at evaluation point $j$ performs worse than base case in at least one NBS scenario (there is at least one $k$ such that $y_{j,k} - a_{j,k} > 0$), the first term is activated in order to minimize the square of maximum inferiority.
- If the new plan at evaluation point $j$ performs better than base case in all NBS scenario, maximize the square root of the minimum improvement.
- Like the single scenario objective function, the square function in the first term gives very high weights to high degradations to force them down as much as possible as a first priority.
- The square root function in the second term gives more weight to small improvements and less weight to large improvements. As such, increasing smaller improvements is of priority over larger improvements.
- Due to the inherent difference of square and square root functions (and as the improvements/degradations, $y_{j,k} - a_{j,k}$, can only be integer), weights given to degradations are much higher that weights given to improvements making the optimization primarily focus on removing/improving the degradations.
- The square and square root functions in the objective function help the optimizer balances the benefits and costs across all evaluation points as described in Section 2.5.2.

A total of eight different NBS scenarios were identified by David Fay of Environment Canada as being diverse in terms of generating a range of high and low lake levels overall as well as differentially across the Great Lakes. These eight NBS scenarios are described in more detail in Appendix 2.

2.5.4 Constraints in the Optimization Formulation

Beside the implicit constraints associated with the routing in the simulation model, it was necessary to consider bound constraints on the decision variables (i.e., rule curve parameters) as opposed to searching in an infinite space. Our selected optimization algorithm requires bounds on decision variable values and appropriately sized bound constraints (i.e., not overly conservative and thus too wide) can substantially enhance the computational efficiency of optimization. For the $b$ and $g$ parameter in component 1 and component 2, lower and upper bounds were selected by trial and error on the basis of average and standard deviation of associated factors (inputs) to the rule curves. Parameters $a$, $c$, and $d$ were assumed to be proportional to $b$ and vary in the ranges of $[2b - 10b]$, $[0.5b - 1.5b]$, and $[2c - 10c]$, respectively. Parameter $h$ was also assumed proportional to $g$ and to vary in $[0.5g - 1.5g]$. Moreover, as stated earlier, the $a$ parameter in component 3 can vary in the range of zero and one.
As mentioned in 2.4.1, the release values calculated from equation (1) is a number in the range of zero and one and should be transformed into an allowable range in units of m$^3$/s. The allowable release range from Lake Superior is set at 280 m$^3$/s to 4010 m$^3$/s for the summer season and 1150 m$^3$/s to 2410 m$^3$/s for winter based on physical limits for the existing structures on the St. Marys River. The release range for Lake MH and Lake Erie is assumed to be 200 m$^3$/s to 12000 m$^3$/s for both seasons. These ranges allow for the biggest possible impact of regulation (least constrained problem). The allowable release range for Lake Ontario is 4810 m$^3$/s to 10010 m$^3$/s for the shipping season and 4360 m$^3$/s to 9490 m$^3$/s for non-shipping season based on the physical and operational constraints of existing structures along the Upper St. Lawrence River. As noted in Section 2.2, the target release values in these ranges are subject to change (resulting in an actual release) to respect the physical/hydraulic constraints of the controlling structures.

2.5.5 Regulation Cost as a Another Objective

Controlling the outflow of Lakes MH and Erie is costly. If the controlled outflow of the lake is higher than the natural connecting channel flow at the same upstream and downstream lake levels then excavation is required to increase the conveyance capacity of the connecting channel. If the controlled outflow is less than the natural connecting channel flow then a structure is required to hold back the water on the lake.

Structural and excavation costs were estimated from cost estimates in the Levels Reference Study, Annex 3 (Working Committee 3, LRS, 1993) pg. 6-50 and updated to 2010 dollars in Appendix B of Bruxer and Carlson (2010). It is assumed that a control structure on St. Clair River to hold back the water on Lake MH costs $513.1 million and a control structure on Niagara River to hold back the water on Lake Erie costs $533.2 million. These structural costs are assumed constant for all degrees of flow reduction relative to natural connecting channel flow and for any range of lake levels. Table 2 and Table 3 report the excavation costs as a function of increased flow in the St. Clair River and Niagara Rivers, respectively.

<table>
<thead>
<tr>
<th>Increase in Q (cms)</th>
<th>Cost (billions 2010 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.00</td>
</tr>
<tr>
<td>142</td>
<td>0.44</td>
</tr>
<tr>
<td>283</td>
<td>0.88</td>
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<tr>
<td>425</td>
<td>1.32</td>
</tr>
<tr>
<td>566</td>
<td>1.76</td>
</tr>
<tr>
<td>850</td>
<td>2.42</td>
</tr>
<tr>
<td>1416</td>
<td>3.96</td>
</tr>
</tbody>
</table>

Table 2. Excavation Cost for Increasing Conveyance of St. Clair River where the Increase in Q is the maximum increase in flow over natural channel flow at the same conditions.
Table 3. Excavation Cost for Increasing Conveyance of Niagara River where the Increase in Q is the maximum increase in flow over natural channel flow at the same conditions.

<table>
<thead>
<tr>
<th>Increase in Q (cms)</th>
<th>Cost (millions 2010 $)</th>
</tr>
</thead>
<tbody>
<tr>
<td>566</td>
<td>88</td>
</tr>
<tr>
<td>566</td>
<td>352</td>
</tr>
<tr>
<td>651</td>
<td>528</td>
</tr>
<tr>
<td>736</td>
<td>616</td>
</tr>
<tr>
<td>850</td>
<td>704</td>
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<td>991</td>
<td>792</td>
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<tr>
<td>1303</td>
<td>880</td>
</tr>
<tr>
<td>1642</td>
<td>968</td>
</tr>
<tr>
<td>2265</td>
<td>1012</td>
</tr>
</tbody>
</table>

The excavation cost can be estimated by interpolating (or extrapolation if required) the largest “Increase in Q” of the St. Clair River from Table 2 and from Table 3 for the Niagara River. Summation of these two costs in addition to the cost of control structures yields an estimate of the total cost of regulation associated with controlling the outflow of Lakes MH and Erie. Equation (22) below provides the total cost estimation.

Cost Objective

\[
\begin{align*}
\text{Cost Objective} & = f_{SCR}^{\text{exc}} \left( \max_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right)_{SCR} \right) + f_{NiR}^{\text{exc}} \left( \max_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right)_{NiR} \right) \\
& + f_{SCR}^{\text{str}} \left( \min_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right)_{SCR} \right) \\
& + f_{NiR}^{\text{str}} \left( \min_t \left( Q_{\text{rule}}^t - Q_{\text{natural}}^t, 0 \right)_{NiR} \right) 
\end{align*}
\]  

(22)

where \( f \) is the cost function, superscripts \( \text{exc} \) and \( \text{str} \) distinguish between excavation cost and cost of building a structure to hold back the water, respectively, and subscripts \( SCR \) and \( NiR \) denote St. Clair and Niagara Rivers respectively. Here, \( f_{\text{exc}} \) is calculated by piecewise linear interpolation (extrapolation if required) of the maximum positive difference between connecting channel flow generated by the rule curve, \( Q_{\text{rule}}^t \), and natural flow in the connecting channel \( Q_{\text{natural}}^t \), in each simulation time step \( t \) (one day in our analysis) from Table 2 or Table 3. \( Q_{\text{natural}}^t \) is calculated by the open-water lake-to-lake stage-fall-discharge equations that describe the current conveyance of the St. Clair and Niagara Rivers in CGLRRM (see Appendix 2 for equations). Also, \( f_{\text{str}} \) is a step function for both SCR and NiR that assigns a constant structural cost ($513.1 million for SCR and $533.2 million for NiR) to any reduction in rule curve based flow relative to the natural flow (negative flow difference in the last two terms of the equation).

The cost estimates above are based on a specific set of alternative structures and a limited range of connecting channel flow changes. The magnitude of flow changes considered in this report as are not constrained, and as results will show, are well beyond the range of conditions considered when these costs were estimated in the Levels Reference Study (Levels Reference Study Board,
Furthermore, the cost estimates for excavation of the Niagara River were noted in Annex 3 of the Levels Reference Study (Working Committee 3, LRS, 1993) to be likely inaccurate for the highest flow changes. Given these factors, it seems clear that any cost estimates based on the above sources are rough and are almost certainly underestimated if extrapolation beyond the data points in Table 2 and Table 3 is required.
3 Results

3.1 Single-objective optimization for frequency-based objective

UW plan with four control points was optimized using two versions of the objective function and subject to the constraints described in Sections 2.5.4. In the first version, the UW plan was optimized with only six evaluation points (Lake Superior, Lake MH, Lake St. Clair, Lake Erie, Lake Ontario, and Iroquois + Saunders headwaters) in the objective function. Note that the Iroquois and Saunders locations are combined to define one evaluation point with the same weight or importance as each upstream lake. In this first version, the Lower St. Lawrence River evaluation points in Montreal (Pointe Claire and Jetty1) are left out of the objective to calculate the maximum possible benefit of the UW plan. With this approach, we roughly assume that structures could be built downstream of Montreal to completely mitigate water level extremes on the Lower St. Lawrence. In the second version of the objective function, the evaluation point associated with the Lower St. Lawrence River at Montreal (Pointe Claire + Jetty1) was also included in the objective function to evaluate the extent to which extreme levels at Montreal could be mitigated by upstream control structures only and to determine the associated impacts on performance at upstream evaluation points. Note that the Pointe Claire and Jetty1 locations are combined to define one evaluation point with the same weight or importance as each upstream lake.

The above two single objective optimization problems involve optimizing 78 rule curve parameters and the evaluation of each solution requires a computationally intensive model simulating the entire Great Lakes – St. Lawrence system over multiple NBS scenarios. Each solution evaluation via this model (i.e., system simulation over the eight NBS scenarios) takes about 2.7 minutes to execute on a 2.8GHz Intel Pentium processor with 2 GB of RAM and running the Windows XP operating system. Thus, a highly efficient optimization algorithm, Dynamically Dimensioned Search (DDS) developed by Tolson and Shoemaker (2007), which has been designed for optimization problems with many decision variables and a limited number of objective function (solution) evaluations, was used to solve the optimization problem. Through preliminary testing, it was recognized that DDS requires at least 3,000 function evaluations (solution evaluations) in this experiment to converge to a desirable region of attraction; therefore, an optimization trial with DDS takes about 5.5 days (~3,000 * 2.7 minutes) of serial computation. To further improve the computational efficiency, the model pre-emption strategy developed by Razavi et al. (2010) was also used in conjunction with the DDS algorithm. The pre-emption strategy was able to reduce the computational time by almost 50 percent.

As the optimization problem formulated here is very complex and multi-modal, both single objective optimization problems were solved using six independent optimization trials conducted with different random seeds to try to ensure that the final best solution out of these six would be reasonable good quality solution (or at least not a single unlucky or poor result from the algorithm). In order to fine-tune the final solutions found using DDS (which are not necessarily exact local optima), the pattern search optimization algorithm (Torczon, 1997) starting from the
best DDS solutions was applied. However, initial results showed that pattern search could not noticeably enhance the quality of DDS solutions suggesting that the solutions found with DDS were all very close to local minima. Therefore, further DDS results presented here were not refined with pattern search.

Figure 3 shows the comparative results of UW plan using the first version of the objective (Lower St. Lawrence River at Montreal evaluation points not considered) optimized through the multi-NBS scenario optimization framework over the eight NBS scenarios. The performance of the UW plan versus the base case plan for each NBS scenario is shown in eight individual subplots in Figure 3. For example, the first subplot in Figure 3 shows the frequency of going beyond the historical simulated extremes at the six evaluation points considered in the objective function for each plan (two series). Note that the frequency of going beyond the historical simulated extremes is also called the frequency of violation in this report and this frequency is the ratio of the number of months when the average level goes beyond the monthly historical simulated extremes over the total number of months in that scenario. As can be seen in Figure 3, the UW plan substantially outperforms the base case at all evaluation points and in all scenarios.

Table 4 also reports the improvement of UW plan over base case (in percent) at different evaluation points under different NBS scenarios. The percent improvement was calculated through the following equation:

\[
\text{Improvement(\%)} = \frac{\text{base case violation frequency} - \text{UW plan violation frequency}}{\text{base case violation frequency}} \times 100
\]

Overall observations from Figure 3 and Table 4 can be summarized as follows:

- Overall improvement (averaged) at all evaluation points and in all NBS scenarios is 68\% for the UW plan.
- Improvement at Lake Erie is 100\% suggesting that the UW plan can completely keep Lake Erie level within the range of extremes simulated for the historical period.
- Improvement at Lake Ontario is the smallest of all evaluation points (35\% averaged over all NBS scenarios).
Figure 3- Performance of UW plan (Lower St. Lawrence River at Montreal evaluation points not considered in objective function) over the eight NBS scenarios in comparison with base case.
<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Avg.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>89%</td>
<td>82%</td>
<td>79%</td>
<td>80%</td>
<td>87%</td>
<td>70%</td>
<td>100%</td>
<td>100%</td>
<td>86%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>50%</td>
<td>78%</td>
<td>58%</td>
<td>93%</td>
<td>43%</td>
<td>56%</td>
<td>49%</td>
<td>66%</td>
<td>62%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>73%</td>
<td>30%</td>
<td>80%</td>
<td>23%</td>
<td>72%</td>
<td>19%</td>
<td>90%</td>
<td>69%</td>
<td>57%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>62%</td>
<td>-17%²</td>
<td>55%</td>
<td>22%</td>
<td>22%</td>
<td>24%</td>
<td>80%</td>
<td>30%</td>
<td>35%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>86%</td>
<td>50%</td>
<td>49%</td>
<td>50%</td>
<td>60%</td>
<td>40%</td>
<td>100%</td>
<td>100%</td>
<td>67%</td>
</tr>
<tr>
<td>Avg.</td>
<td>77%</td>
<td>54%</td>
<td>70%</td>
<td>62%</td>
<td>64%</td>
<td>52%</td>
<td>87%</td>
<td>77%</td>
<td>68%</td>
</tr>
</tbody>
</table>

1. % Improvement = 100[Base Case \(\text{extreme violation count}\) - UW \(\text{extreme violation count}\)]/ Base Case \(\text{extreme violation count}\)
2. For this result, the number of violation in base case is 6 (frequency=0.71%) while the same figure in UW plan is 7 (frequency= 0.83%)

Figure 4 compares the above UW plan results (first version of the objective leaving Montreal out of the objective function) to the UW plan results obtained when including Montreal in the objective function. Note the new evaluation point at Montreal (Pointe Claire and Jetty1) now in each of the subplots in Figure 4. Table 5 reports the improvement of UW plan over base case (in percent) at different evaluation points under different NBS scenarios and can be compared directly with Table 4. Observations from Figure 4 and Table 5 can be summarized as follows:

- Performance at Montreal is approximately the same or slightly worse than the base case across the eight NBS scenarios when it is included in the objective function.
- Managing for Montreal using only the upstream four control points causes some degradation in UW plan performance at upstream evaluation points (average of 34% improvement compared to 68% in Table 4 for upstream evaluation points).
- Most degradation of upstream performance after considering Montreal in the objective is observed at Lake MH and then at Lake Superior. This observation might be solution specific, and there may be other solutions to the same optimization problem being less affected by considering Montreal.
- UW plan considering Montreal can still improve upon base case in all evaluation points upstream of Saunders in all scenarios except at IHW+SHW in scenarios 4 and 6. Further optimization may solve this problem and might be able to show equivalent performance at Montreal compared to the base case. Optimization on this problem is ongoing.
Figure 4- Performance of UW plan with and without Montreal (Lower St. Lawrence) in the objective function for the eight NBS scenarios in comparison with base case.
Table 5- Improvement\(^1\) of UW plan at each evaluation point for each NBS scenario relative to base case considering Montreal in the objective function.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>82.54%</td>
<td>63.33%</td>
<td>65.75%</td>
<td>58.39%</td>
<td>73.33%</td>
<td>3.70%</td>
<td>28.57%</td>
<td>92.31%</td>
<td>58.49%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>39.78%</td>
<td>65.00%</td>
<td>11.54%</td>
<td>26.83%</td>
<td>26.02%</td>
<td>8.06%</td>
<td>3.02%</td>
<td>41.86%</td>
<td>27.76%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>74.74%</td>
<td>63.64%</td>
<td>78.82%</td>
<td>52.22%</td>
<td>78.56%</td>
<td>47.37%</td>
<td>82.14%</td>
<td>86.51%</td>
<td>70.12%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>100.00%</td>
<td>100.00%</td>
<td>99.12%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>100.00%</td>
<td>99.89%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>69.75%</td>
<td>-66.67%</td>
<td>60.76%</td>
<td>-22.22%</td>
<td>63.01%</td>
<td>21.57%</td>
<td>71.52%</td>
<td>90.00%</td>
<td>35.96%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>76.79%</td>
<td>25.00%</td>
<td>60.27%</td>
<td>-800.00%(^2)</td>
<td>77.78%</td>
<td>-320.00%(^3)</td>
<td>94.20%</td>
<td>100.00%</td>
<td>-85.74%</td>
</tr>
<tr>
<td>Average</td>
<td>74%</td>
<td>42%</td>
<td>63%</td>
<td>-97%</td>
<td>69%</td>
<td>-23%</td>
<td>63%</td>
<td>85%</td>
<td>34%</td>
</tr>
</tbody>
</table>

1. \(\% \text{ Improvement} = 100 \left( \frac{\text{Base Case}_{\text{extreme violation count}} - \text{UW}_{\text{extreme violation count}}}{\text{Base Case}_{\text{extreme violation count}}} \right)\)
2. Inflated due to very small number of extreme violation counts (18 for UW plan versus 2 in base case).
3. Inflated due to very small number of extreme violation counts (21 for UW plan versus 5 in base case).

3.2 Validation performance of UW plan and Base Case over the full 50,000 year LOSL NBS sequence

The base case and UW plans (without Montreal evaluation points in the objective function) were simulated for the entire 50,000 year stochastic sample with initial lake levels set to the year 1900 levels. Note that the UW plan including Montreal in the objective function was not evaluated here since it is still being refined by further optimization. Statistics on plan performance with regards to the frequency of violating monthly extreme lake levels, the magnitudes of these violations, the distribution of lake levels and the distribution connecting channel flows were analyzed. These are summarized in the following Tables and Figures.

Table 6 reports the frequency of violating extremes (upper and lower, as defined in Table 1) as well as the frequencies associated to lower extreme and upper extreme separately. The total violation frequencies are substantially lower for the UW plan compared to the base case except for Lake St. Clair. Lake St. Clair outflows are not controlled and as such Lake St. Clair levels can fluctuate greatly as a result of the controlled Lake MH outflows. These higher frequencies associated to the UW plan largely violate the upper extreme lake levels (frequencies are lower than base case for violating lower extremes) and this is expected as Lake St Clair is not sufficiently large to dampen the very large flows now possible from Lake MH that are generated by the UW plan. Overall, for the base case, 6.2% of months are simulated such that at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes. This number is reduced to 3.8% for the UW plan and the majority of those violations are due to Lake St. Clair as can be seen in Table 6. In fact, if Lake St. Clair is ignored, the UW plan simulates 1.0% of months such that at least one evaluation point from Superior through Saunders HW is violating the 1900-2008 period extremes compared to 5.5% of months in the base case.
Table 6- Frequency of violating the monthly simulated extremes for the historical period (1900-2008) based on simulation of full 50,000 year stochastic LOSL NBS data set.

<table>
<thead>
<tr>
<th></th>
<th>UW Plan</th>
<th></th>
<th>Base Case</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency of Violating Extremes</td>
<td>Frequency of Violating Upper Extreme</td>
<td>Frequency of Violating Lower Extreme</td>
<td>Frequency of Violating Upper Extreme</td>
</tr>
<tr>
<td>Lake Superior</td>
<td>0.16%</td>
<td>0.12%</td>
<td>0.04%</td>
<td>2.31%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>0.67%</td>
<td>0.08%</td>
<td>0.60%</td>
<td>2.59%</td>
</tr>
<tr>
<td>Lake St Clair</td>
<td>2.99%</td>
<td>2.33%</td>
<td>0.66%</td>
<td>2.75%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>1.98%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>0.35%</td>
<td>0.16%</td>
<td>0.19%</td>
<td>0.87%</td>
</tr>
<tr>
<td>IHW</td>
<td>0.03%</td>
<td>0.01%</td>
<td>0.02%</td>
<td>0.64%</td>
</tr>
<tr>
<td>SHW</td>
<td>0.09%</td>
<td>0.00%</td>
<td>0.09%</td>
<td>0.03%</td>
</tr>
</tbody>
</table>

Figure 5 and Figure 6 show the histograms of the magnitude of extreme violations at all evaluation points. As an example, Lake Superior when operated through UW plan goes beyond the upper extremes (see upper left subplot in Figure 5) by 0-0.05 metres in 397 of the 600,000 (50,000 * 12) months in the simulation, while in the base case it goes beyond the upper extreme by 0-0.05 metres in 3567 months. For SHW, since we did not assume an upper extreme as discussed in Section 2.5.1, this figure only has the histogram plot for SHW when the water level violates lower extreme. Figure 5 and Figure 6 show the UW plan generates substantially lower numbers in each bin for all evaluation points, except for Lake St. Clair when violating the upper extreme and Saunders Headwater lower extreme. For example, the UW plan generates no extreme violations for Lake Erie and very few violations on Lake Superior and IHW. Overall, these histograms show that in addition to reducing the frequency of extreme violations, the magnitudes of the extreme violations are also substantially reduced by the UW plan (despite the fact that these magnitudes are not considered in the optimization formulation).

Figure 7 compares the UW plan to the base case in terms of the percentiles of water levels at all evaluation points (Lake Superior through Saunders headwaters) while Figure 8 compares the UW plan to the base case in terms of the percentiles of connecting channel flows (St. Marys River through Lake Ontario outflow). In Figure 7, the UW plan clearly compresses the range of water levels simulated over the 50,000 year period for all evaluation points except for Lake St. Clair and Saunders headwater. In these two exceptions, the range of lake levels are approximately the same in the UW plan and the base case and both Lake St. Clair high and low levels are higher relative to base case. Figure 8 shows that the compressed water level ranges in the UW plan appear to require expanding the range of flows experienced in the connecting channels (except for Lake Ontario outflow).
Figure 5. Histograms of monthly average extreme violation magnitudes for Lake Superior through Lake Erie evaluation points for the 50,000 year simulation under the UW and base case plans – total number of time intervals (months) is 600,000.
Figure 6. Histograms of monthly average extreme violation magnitudes for Lake Ontario through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the UW and base case plans – total number of time intervals (months) is 600,000.
Figure 7. Select quantiles from the frequency distribution of monthly average water levels for Lake Superior through Saunders Headwater (SHW) evaluation points for the 50,000 year simulation under the UW and base case plans.
Figure 8. Select quantiles from the frequency distribution of monthly average connecting channel flows for the St Mary's River through Lake Ontario Outflow for the 50,000 year simulation under the UW and base case plans. Note that there is no structure controlling Lake St. Clair (Detroit River) outflows in the UW plan.

3.3 Multi-objective optimization results for frequency-based objective and regulation costs associated with St. Clair and Niagara River control points

The UW plan from Section 3.1 (not including the lower St. Lawrence in the objective function) was derived ignoring the implications of regulation costs as detailed in Section 2.5.5. When Equation (22) is used to compute the costs of regulation across all eight NBS scenarios (all eight scenarios are treated as one long scenario for the purposes of computing maximum and minimum differences over all time steps in this equation), the regulation cost is estimated at just under $30 billion. A multi-objective (bi-objective) optimization model to minimize the previous frequency-based objective and minimize the cost objective was solved to assess the tradeoff between improved system performance (frequency-based objective) and regulation costs. In other words, the range of system performance improvements costing under $30 billion was of interest to determine.
The above bi-objective optimization problem is solved by the Pareto Archived Dynamically Dimensioned Search (PA-DDS) optimization algorithm developed by Asadzadeh and Tolson (2009). PA-DDS is a heuristic multi-objective optimization algorithm based on the single objective DDS algorithm introduced by Tolson and Shoemaker (2007) for computationally intensive hydrologic model calibration problems. PA-DDS conducts the search in the exact same way as DDS does; moreover, it archives all the non-dominated (tradeoff) solutions during the search. Also, whenever PA-DDS cannot find a new non-dominated solution, it uses the crowding distance metric (Deb et al. 2000) to select a current non-dominated solution as the center of the search neighbourhood. PA-DDS inherits the parsimonious characteristic of DDS, so it has only one algorithm parameter and all PA-DDS runs apply the same (default) algorithm parameter value ($r=0.2$).

PA-DDS was applied iteratively (sequential optimization trials) to solve this bi-objective problem in order to continually improve the tradeoff approximation and focus the algorithm on the most relevant parts of the tradeoff. The PA-DDS algorithm was initially seeded with the results of multiple single objective optimization trials (different solutions) from the analyses of Section 3.1. Results from the first PA-DDS optimization trial showed two distinct clusters of solutions on the tradeoff. One cluster close to the best known frequency-based objective function value (approximately -20) but almost $30$ billion in cost and another cluster with a lower cost (around $5$-$6$ billion) but very large frequency based objective function value (more than 1000). Additional PA-DDS optimization trials demonstrated that the frequency-based objective of the second cluster of solutions could be substantially reduced with only small relative increases in their cost. Therefore, a further suite of experiments was designed to sequentially solve the problem while focusing on only these two clusters of solutions and avoiding altogether the evaluation of solutions with poor frequency-based objective values solutions and thus costs lower than $5$ billion. In these final experiments, a multi-objective pre-emption concept similar to that in Razavi et al. (2009) is applied to reject the complete evaluation of solutions with an unacceptably poor frequency-based objective function. This requires defining a threshold for rejecting the complete evaluation of solutions based on the frequency objective and the first pre-emption enabled PA-DDS optimization trials used a value of 1000 for this threshold. The threshold was reduced sequentially in later trials to a final value of 100.

The results of the iterative PA-DDS approach described above (more than ten sequential optimization trials, as well as multiple independent trials of PA-DDS, totaling more than 30,000 evaluated solutions) were aggregated together to determine the final set of non-dominated or tradeoff solutions and this set of solutions in depicted in Figure 9. There are two distinct clusters of solutions on the tradeoff separated by a very large difference in regulation costs (a difference of approximately $20$ billion). It is not currently clear why there is such a distinct difference in cost or whether this difference actually exists in the true set of tradeoff solutions (remember, PA-DDS is heuristic and can only be expected to approximate the true tradeoff like all other applicable multi-objective optimization algorithms). Computational experiments with PA-DDS
(beyond results reported here) continue to refine the tradeoff approximation in order to determine if other solutions exist with costs ranging from $6 to $25 billion dollars and a corresponding frequency-based objective between -30 and -10.

![Graph](image)

**Figure 9.** Estimated tradeoff between the frequency-based objective function (Equation 21) and the regulation costs associated with controlling flow on the Niagara and St. Clair Rivers (Equation 22). Note that a negative value of the frequency-based objective implies that the frequency of violating extremes is improved or equal to the base case everywhere and for all eight NBS scenarios. The blue and green coloured solutions are compared in Figure 10 and Table 7 and Table 8.

The $29.6 billion solution (blue triangle) in Figure 9 clearly uses extrapolated excavation costs beyond the data in Table 2 and Table 3 and these are caused by the maximum differences in flows between the current conveyance regime and the UW plan actual release of 9450 cms for the St. Clair River and 6520 cms for the Niagara River flow. Analyses are being conducted to determine if such differences are even necessary to improve the frequency-based objective and if not, to determine if such high differences can be avoided by adjusting the UW plan. These analyses may be unnecessary given the quality of the $8.5 billion solution which also has a very good negative frequency-based objective value. Further optimization with PA-DDS may improve the tradeoff (push it to the left) even further. Note also that PA-DDS slightly improved the best frequency-based UW plan (without the lower St. Lawrence in the objective) depicted and analyzed extensively in Section 3.1 and 3.2.

The $29.6 billion (blue triangle) and $8.5 billion (green diamond) solutions in Figure 9 are two of the most interesting solutions from a decision-making standpoint and as such, the relevant question is how to compare quality on the frequency-based objective function scale (which does not have interpretable units). Note that the $29.6 billion solution in Figure 9 is a different, slightly less expensive solution than the $30 billion solution from Section 3.1 and discussed at the beginning of this Section. A negative value of the frequency-based objective generally
implies that the frequency of violating extremes is improved or equal to the base case everywhere and for all eight NBS scenarios while a positive value implies there is at least one evaluation point in at least one NBS scenario which performs worse than the base case. How much worse, is not clear based on the value of the objective function. Therefore, the blue and green solutions are compared in terms of the raw frequencies of going beyond the simulated historical period extremes for each evaluation point and each NBS scenario in Figure 10.

The UW plan costing $29.6 billion improved upon or equaled the base case performance at all evaluation points in all NBS scenarios. Except for four instances at Lake Ontario and IHW+SHW, where the $8.5 billion solution is just marginally worse than the base case, the $8.5 billion solution also improved upon or equaled the base case performance at all evaluation points in all NBS scenarios. Other than those differences, it would appear that these two solutions are very comparable in terms of overall performance. Given the minor degraded performance relative to the base case in four instances (an increase of at most 0.2% frequency of extreme violations relative to the base case), it seems clear that decision-makers would not consider spending an additional $21 billion dollars to achieve the absolute best frequency-based objective function. The exact multi-lake rule curve equations defining the $8.5 billion solution are detailed in Appendix 4. Future results and analysis after more consultation with the IUGLS study board will focus on a careful analysis of one or more of the solutions in the lower cost cluster of solutions.

Table 7 and Table 8 report the improvement of the $29.6 billion and $8.5 billion UW plans, respectively, over the base case (in percent) at different evaluation points under different NBS scenarios. Overall, the more expensive plan generates a higher average improvement across all NBS scenario and evaluation point combinations (73% versus 52%). Table 7 shows that this UW plan now achieves (unlike Table 4) the overall objective of decreasing the frequency of extreme lake levels for all NBS scenarios at all evaluation points. This overall objective is not achieved by the lower cost ($8.5 billion) UW plan as shown by the few negative numbers (implying worse UW plan performance than the base case) for Lake Ontario and IHW-SH evaluation points in Table 8. However, these negative numbers are somewhat misleading since the violation frequencies involved in their calculation are so small (all less than 1%). Table 7 and Table 8 show that on average, the violation frequencies on Lake Superior and Lake Erie are all substantially improved (reduced by 77% or more) while Lake MH derives smaller benefits from these multi-lake regulation plans (violation frequency reduced by approximately 40%). However, Table 4 shows that for a different multi-lake regulation plan (with nearly the same frequency-based objective function value), Lake MH violation frequency can be reduced by approximately 62%.

The comparison of the various solutions in Table 7 and Table 8 (and Table 4) highlights that a variety of reasonable quality solutions (multi-lake regulation plans) exist for a range of estimated costs. Other reasonable low cost solutions also exist based on results in Figure 9. These different solutions will differ in terms of the relative improvements observed at each evaluation point and will differ in other respects (i.e., connecting channel flow or lake level variability over
time). Therefore, more analysis and decision-maker input would be required to identify a single solution that was most appropriate for continued consideration in multi-lake regulation.

Figure 10. Comparison of blue and green tradeoff solutions from Figure 9 in terms of violation frequencies for each NBS scenario from Lake Superior through to Iroquois + Saunders Headwater evaluation points.
Table 7- Improvement of $29.6 billion UW plan at each evaluation point for each NBS scenario relative to base case.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>94%</td>
<td>82%</td>
<td>84%</td>
<td>73%</td>
<td>100%</td>
<td>65%</td>
<td>100%</td>
<td>100%</td>
<td>87%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>26%</td>
<td>53%</td>
<td>44%</td>
<td>82%</td>
<td>23%</td>
<td>37%</td>
<td>15%</td>
<td>40%</td>
<td>40%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>75%</td>
<td>43%</td>
<td>84%</td>
<td>60%</td>
<td>70%</td>
<td>46%</td>
<td>90%</td>
<td>75%</td>
<td>68%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>89%</td>
<td>17%</td>
<td>77%</td>
<td>22%</td>
<td>66%</td>
<td>25%</td>
<td>95%</td>
<td>90%</td>
<td>60%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>100%</td>
<td>50%</td>
<td>93%</td>
<td>100%</td>
<td>89%</td>
<td>40%</td>
<td>100%</td>
<td>100%</td>
<td>84%</td>
</tr>
<tr>
<td>Average</td>
<td>81%</td>
<td>57%</td>
<td>80%</td>
<td>73%</td>
<td>75%</td>
<td>52%</td>
<td>83%</td>
<td>84%</td>
<td>73%</td>
</tr>
</tbody>
</table>

1. $\%$ Improvement = \[\frac{\text{Base Case}_{\text{extreme violation count}} - \text{UW}_{\text{extreme violation count}}}{\text{Base Case}_{\text{extreme violation count}}} \] * 100

Table 8- Improvement of $8.5 billion UW plan at each evaluation point for each NBS scenario relative to base case.

<table>
<thead>
<tr>
<th>Evaluation Point</th>
<th>Scenario 1</th>
<th>Scenario 2</th>
<th>Scenario 3</th>
<th>Scenario 4</th>
<th>Scenario 5</th>
<th>Scenario 6</th>
<th>Scenario 7</th>
<th>Scenario 8</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lake Superior</td>
<td>65%</td>
<td>80%</td>
<td>79%</td>
<td>86%</td>
<td>53%</td>
<td>78%</td>
<td>86%</td>
<td>85%</td>
<td>77%</td>
</tr>
<tr>
<td>Lake MH</td>
<td>39%</td>
<td>48%</td>
<td>54%</td>
<td>13%</td>
<td>41%</td>
<td>26%</td>
<td>58%</td>
<td>50%</td>
<td>41%</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>45%</td>
<td>48%</td>
<td>60%</td>
<td>68%</td>
<td>39%</td>
<td>47%</td>
<td>62%</td>
<td>53%</td>
<td>53%</td>
</tr>
<tr>
<td>Lake Erie</td>
<td>96%</td>
<td>89%</td>
<td>95%</td>
<td>97%</td>
<td>93%</td>
<td>87%</td>
<td>100%</td>
<td>100%</td>
<td>95%</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>57%</td>
<td>-17%</td>
<td>52%</td>
<td>17%</td>
<td>19%</td>
<td>8%</td>
<td>65%</td>
<td>40%</td>
<td>30%</td>
</tr>
<tr>
<td>IHW+SHW</td>
<td>46%</td>
<td>63%</td>
<td>29%</td>
<td>-150%$^2$</td>
<td>-4%</td>
<td>-40%</td>
<td>90%</td>
<td>100%</td>
<td>17%</td>
</tr>
<tr>
<td>Average</td>
<td>58%</td>
<td>52%</td>
<td>62%</td>
<td>22%</td>
<td>40%</td>
<td>34%</td>
<td>77%</td>
<td>71%</td>
<td>52%</td>
</tr>
</tbody>
</table>

1. $\%$ Improvement = \[\frac{\text{Base Case}_{\text{extreme violation count}} - \text{UW}_{\text{extreme violation count}}}{\text{Base Case}_{\text{extreme violation count}}} \] * 100

2. Inflated due to very small number of extreme violation counts (5 for UW plan versus 2 in base case).
4 Future Work

The following list summarizes a list of suggested additional analyses to be conducted:

- Continued computational effort to improve solution quality through further optimization. This effort will likely focus on the solutions estimated to cost below about $10 billion and will be conducted with and without the Lower St. Lawrence evaluation points in the objective function.

- Initial results considering only the two existing regulation structures with new optimal release policies were generated but they utilized assumptions that were inconsistent with the four control point formulation assumptions and thus invalidated any comparisons. New rule curves for the two existing regulation structures will be optimized with equivalent assumptions and then compared directly to results in this report in order to evaluate the added value associated with two new structures.

- Generate the tradeoff (more multi-objective optimization) between regulation costs and the frequency-based objective when the Lower St. Lawrence (Montreal) is included in the objective function.

- Evaluate various UW plan performance (and the base case plan) against a climate change scenario from the LOSLR study (International Lake Ontario-St. Lawrence River Study Board, 2006).

5 Conclusions

Four point multi-lake regulation (regulating Lakes Superior, Michigan-Huron, Erie and Ontario) on the Great Lakes can simultaneously reduce the frequency of extreme water levels and reduce the magnitude of extreme violations across multiple extreme NBS scenarios and at all evaluation points in the system relative to the base case regulation strategy. This finding is specific to the Lake Superior through Upper St. Lawrence evaluation points and assumes structures are constructed on the Lower St. Lawrence that can completely mitigate extreme water levels in the Lower St. Lawrence River. Without structures on the Lower St. Lawrence, a four point multi-lake regulation plan that mitigates extreme water levels in the Lower St. Lawrence in addition to all other evaluation points has been developed but it is less effective in the Upper Lakes (particularly Lake MH and Lake Superior). Additional work is needed to evaluate whether multi-lake regulation is feasible without considering structures on the Lower St. Lawrence River.

Considering only Lake Superior through Upper St. Lawrence evaluation points (ignoring impacts on the Lower St. Lawrence), the current approximation of the tradeoff between regulation costs (associated only with controlling Lake MH and Lake Erie outflow) and the corresponding reduction in the frequency of extreme lake level violations shows the regulation costs range from approximately $6 to $30 billion dollars. These costs do not consider mitigation costs that may be required downstream on the lower St. Lawrence River. However, it is apparent that reasonable quality solutions exist for an estimated $8.5 billion and increasing costs to $30 billion do not
generate large decreases in the frequency of violations. Continued multi-objective optimization may lead to even more desirable solutions with costs around $8.5 billion.

Continued analysis and consultation with the IUGLS board should reveal the scope of further exploratory study on multi-lake regulation. For example, evaluating the $8.5 billion UW plan performance with one or more climate change scenarios from the LOSL study (International Lake Ontario-St. Lawrence River Study Board, 2006) should help determine the feasibility of multi-lake regulation on the Great Lakes without considering structures on the Lower St. Lawrence River. Additional work to determine the amount of extreme lake level mitigation with only new rule curves at the two current structures locations (Lake Superior and Lake Ontario outflow) is ongoing. Given the approximate nature of the regulation cost estimates, further exploratory study should give consideration to revising and updating these cost estimates for changes to the St. Clair River and Niagara River channels.
References


7 Appendices

7.1 Appendix 1 – Routing equations.

Set of routing equations to solve for determining Lake St. Clair and Lake Erie water levels and Detroit River flows.

\[
\begin{align*}
NBS_3 + D_3 - QR_2 - QO_3 + k_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^b_2 &= \\
\{\begin{array}{c}
-0.5k_2b_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^{(b_2 - 1)} - 0.5k_2a_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) (a_2 - 1) (z_2 - z_3)^{b_2} \\
\end{array} + \\
\{0.5k_2b_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^{(b_2 - 1)} - 0.5k_2a_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) (a_2 - 1) (z_2 - z_3)^{b_2} + sa_3/\Delta t\} \Delta Z_2 \\
\end{align*}
\]

\[
\begin{align*}
NBS_2 + D_2 + QR_2 + QO_1 - k_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^b_2 &= \\
\{0.5k_2b_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^{(b_2 - 1)} + 0.5k_2a_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) (a_2 - 1) (z_2 - z_3)^{b_2} + sa_2/\Delta t\} \Delta Z_2 \\
+ \\
\{0.5k_2b_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) a_2(z_2 - z_3)^{(b_2 - 1)} + 0.5k_2a_2(\varphi_2 z_2 + (1 - \varphi_2)z_3 - y_m_2) (a_2 - 1) (z_2 - z_3)^{b_2} \} \Delta Z_3
\end{align*}
\]

Subscripts:
1: Lakes Michigan-Huron
2: Lake St. Clair
3: Lake Erie

Variables:
NBS: Net Basin Supply
QO: Lake outflow
QR: Ice retardation
D: Diversion (+ into the lake, - out of the lake)
A: Lake surface area
Z: Lake elevation
\(\Delta t\): model timestep
\(\varphi\): weighting coefficient (between 0 and 1) for stage-discharge relationship
K: empirical coefficient for stage-discharge relationship
a: empirical coefficient for stage-discharge relationship
b: empirical coefficient for stage-discharge relationship
ym: empirical bottom elevation for stage-discharge relationship
\(\Delta Z\): change in lake surface elevation for \(\Delta t\)
7.2 Appendix 2 – NBS scenarios for multi-lake rule curve development

Selected and described by David Fay of Environment Canada (EC) from the 50,000 year stochastic NBS set produced by Hydro Quebec (Fagherazzi, 2011) for the LOSLR Study (International Lake Ontario-St. Lawrence River Study Board, 2006).

Selection was based on EC simulated 77A levels (from 2009 version of the 77A CGLRRM). Series are 70-80 years long and each one starts at what are close to initial lake level conditions similar to those used in the base case (e.g., 1900). Selected series (referenced to the years from 1 to 50000 in the stochastic NBS set produced by Hydro Quebec (Fagherazzi et al., 2005) are described as follows:

Combined Superior, MH and Erie levels containing max and min annual average

1.  Low: years 35385 to 35454
2.  High: years 5703 to 5772

Combined Superior, MH and Erie levels containing max min 30 year moving average

3.  Low: years 27001 to 27070
4.  High: years 16016 to 16085

Large differences in Sup and MH levels

5.  Sup higher: years 25225 to 25304
6.  MH higher: years 11668 to 11747

Sets from LOSL study (Reduced to 70 years)

7.  Long drought on ONT: 15185-15254

Random set

8.  Random selection 14324-14393
7.3 Appendix 3 – Natural (current) connecting channel flow equations.
Open water lake-to-lake equations for natural connecting channel flows are as follows:

Natural instantaneous flow in St. Clair River (m³/s):

\[ Q_{O_{SCR}} = 186.90 \left( 0.5 Z_{MH} + 0.5 Z_{SC} - 167.0 \right)^{1.51} \left( Z_{MH} - Z_{SC} \right)^{0.36} - \text{Retardation}_{SCR} \]

Natural instantaneous flow in Niagara River (m³/s):

\[ Q_{O_{NIR}} = 558.3 \left( Z_{ER} - 169.86 \right)^{1.6} - \text{Retardation}_{Nia} \]

\( Z_{MH}, Z_{SC} \) and \( Z_{ER} \) are the average lake levels (m) for lakes Michigan-Huron, St. Clair and Erie while the empirical Retardation factors are reductions in flow due to the presence of ice and weeds.
Appendix 4

Complete multi-lake rule curve parameters for the $8.5 billion UW plan

The three tables below detail the exact multi-lake rule curve and can thus be used to simulate the system operation for any NBS scenario. The rule curve equations presented in Section 2.4.1 use these parameters and generate a target release value scaled in the range of zero and one. Transforming this scaled target release value to the original scale outlined in Section 2.5.4 would result in the actual target release.

Table A4-1 Rule Curve Parameters

<table>
<thead>
<tr>
<th>Rule Curve Parameters</th>
<th>Lake Superior</th>
<th>Lake MH</th>
<th>ERI</th>
<th>Lake Ontario</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>winter</td>
<td>Summer</td>
<td>winter</td>
<td>Summer</td>
</tr>
<tr>
<td>a</td>
<td>17.444</td>
<td>24.796</td>
<td>3.57E-12</td>
<td>5.13E-12</td>
</tr>
<tr>
<td>b</td>
<td>2.526</td>
<td>2.878</td>
<td>1.78E-12</td>
<td>2.57E-12</td>
</tr>
<tr>
<td>c</td>
<td>2.366</td>
<td>1.708</td>
<td>1.84E-12</td>
<td>1.30E-12</td>
</tr>
<tr>
<td>d</td>
<td>14.849</td>
<td>5.483</td>
<td>4.41E-12</td>
<td>7.87E-12</td>
</tr>
<tr>
<td>e</td>
<td>0.200</td>
<td>0.191</td>
<td>1.35E+11</td>
<td>1.35E+11</td>
</tr>
<tr>
<td>f</td>
<td>0.167</td>
<td>0.308</td>
<td>1.36E+11</td>
<td>1.35E+11</td>
</tr>
<tr>
<td>g</td>
<td>-1.11E-11</td>
<td>-6.12E-12</td>
<td>-2.81E-12</td>
<td>-6.90E-12</td>
</tr>
<tr>
<td>h</td>
<td>-9.47E-12</td>
<td>-6.85E-12</td>
<td>-1.41E-12</td>
<td>-9.90E-12</td>
</tr>
<tr>
<td>alpha</td>
<td>0.572</td>
<td>0.507</td>
<td>0.923</td>
<td>0.298</td>
</tr>
<tr>
<td>i</td>
<td>0.631</td>
<td>0.887</td>
<td>0.408</td>
<td>0.410</td>
</tr>
</tbody>
</table>

Table A4-2 Monthly averaged lake levels (m) over historical period used for rule curve development

<table>
<thead>
<tr>
<th>Lake Superior</th>
<th>Jan</th>
<th>Feb</th>
<th>Mar</th>
<th>Apr</th>
<th>May</th>
<th>Jun</th>
<th>Jul</th>
<th>Aug</th>
<th>Sep</th>
<th>Oct</th>
<th>Nov</th>
<th>Dec</th>
</tr>
</thead>
<tbody>
<tr>
<td>Jettyle</td>
<td>183.33</td>
<td>183.27</td>
<td>183.24</td>
<td>183.26</td>
<td>183.37</td>
<td>183.45</td>
<td>183.51</td>
<td>183.54</td>
<td>183.54</td>
<td>183.51</td>
<td>183.47</td>
<td>183.41</td>
</tr>
<tr>
<td>Lake MH</td>
<td>176.31</td>
<td>176.29</td>
<td>176.31</td>
<td>176.4</td>
<td>176.49</td>
<td>176.56</td>
<td>176.59</td>
<td>176.57</td>
<td>176.52</td>
<td>176.45</td>
<td>176.39</td>
<td>176.34</td>
</tr>
<tr>
<td>Lake St. Clair</td>
<td>174.84</td>
<td>174.79</td>
<td>174.9</td>
<td>175.04</td>
<td>175.12</td>
<td>175.17</td>
<td>175.19</td>
<td>175.15</td>
<td>175.08</td>
<td>174.99</td>
<td>174.91</td>
<td>174.91</td>
</tr>
<tr>
<td>Lake Ontario</td>
<td>74.56</td>
<td>74.6</td>
<td>74.68</td>
<td>74.88</td>
<td>75.02</td>
<td>75.05</td>
<td>74.99</td>
<td>74.88</td>
<td>74.74</td>
<td>74.61</td>
<td>74.54</td>
<td>74.53</td>
</tr>
</tbody>
</table>
An illustrative example is given below to show how the rule curve formulation and the rule curve parameters above can be used to generate the target release values. Suppose that we want to calculate the Lake MH target release in the first quarter month of September (ice free season) using the developed rule curves.

Given data For Component 1:

Length of the first qm of September = 8 days

\[ Z_{MH} = 176.60 \text{ masl (MH lake level at the beginning of the qm)} \]

\[ RV_{Sup} = 2400 \text{ cms (planned Lake Superior release)} \times 8 \text{ days} \times 24 \text{ hr/day} \times 3600 \text{ s/hr} \]

\[ = 165888 \times 10^4 \text{ m}^3 \]

\[ A_{MH} = 1174 \times 10^8 \text{ m}^2 \text{ (Lake MH area)} \]

\[ \text{avg}Z_{MH} = 176.52 \text{ masl (from Table A3-2)} \]

\[ \text{avg}RV_{Sup} = 2326 \text{ cms (from Table A3-3)} \times 8 \times 24 \times 3600 \text{ s} = 160773.1200 \times 10^4 \text{ m}^3 \]

Using Equation (8) in Section 2.4.1:

\[ \text{UpstreamStorage}(2) = (RV_{Sup} - \text{avg}RV_{Sup}) + A_{MH}(Z_{MH} - \text{avg}Z_{MH})= \]

\[ = (165888 \times 10^4 - 160773.1200 \times 10^4) + 1174 \times 10^8 (176.60 - 176.52) \]

\[ = 9443148800 \text{ m}^3 \]

As \text{UpstreamStorage}(2) > 0, Lake MH is in excess condition, the piece-wise linear function defined in Figure 1 should be used to calculate Component 1:

According to Table A3-1, rule curve parameter \( f \) is 1.35E+11, and as \ 9443148800 < 1.35E+11, Component 1 = 2.57E-12 \times 9443148800 = 0.02426889

where 2.57E-12 is parameter \( b \) from Table A3-1
Given data For Component 2:

\[ Z_{SC} = 174.9 \text{ masl} \]
\[ \text{avg} Z_{SC} = 175.08 \text{ masl} \]
\[ A_{SC} = 11.14 \times 10^8 \text{ m}^2 \text{ (Lake St Clair area)} \]
\[ Z_{Er} = 173.95 \text{ masl} \]
\[ \text{avg} Z_{Er} = 174.16 \text{ masl} \]
\[ A_{ER} = 257 \times 10^8 \text{ m}^2 \text{ (Lake Erie area)} \]

Using Equation (12) in Section 2.4.1:

\[ \text{DownstreamStorage}_\text{im}(2) = A_{SC} (Z_{SC} - \text{avg} Z_{SC}) + A_{ER} (Z_{Er} - \text{avg} Z_{Er}) \]
\[ = 11.14 \times 10^8 (174.9 - 175.08) + 257 \times 10^8 (173.95 - 174.16) \]
\[ = -5597520000 \text{ m}^3 \]

As \text{DownstreamStorage}_\text{im}(2) < 0, the set of Lake St. Clair + Lake Erie is in shortage condition

The piece-wise linear function defined in Figure 1 should be used to calculate Component 2:

Component 2 = -9.90E-12 \times -5597520000 = 0.055415448

where -9.90E-12 is parameter \( h \) from Table A3-1

Given data For Component 3:

\[ A_{ON} = 195 \times 10^8 \text{ m}^2 \text{ (Lake Ontario area)} \]
\[ Z_{ON} = 74.3 \text{ masl} \]
\[ \text{avg} Z_{On} = 74.74 \text{ masl} \]

Using Equation (16) in Section 2.4.1:

\[ \text{DownstreamStorage}_\text{far}(2) = A_{ON} (Z_{ON} - \text{avg} Z_{On}) \]
\[ = 195 \times 10^8 (74.3 - 74.74) \]
\[ = -8580000000 \text{ m}^3 \]
As DownstreamStorage_far(2) < 0, Lake Ontario is in shortage condition. As both DownstreamStorage_im(2) and DownstreamStorage_far(2) are in shortage condition, Component 3 is active and should be calculated through the piece-wise linear function defined in Figure 2 as:

$$\text{Component 3} = 0.298 \times -9.90 \times 10^{-12} \times -8580000000 = 0.025313$$

where 0.298 is parameter \( \alpha \) from Table A3-1

*Given all component values, compute normalized and then real-scale target releases:*

Now, using Equation (6) in Section 2.4.1, the normalized MH target release can be calculated as:

$$\text{Target Release_{normal}} = \text{Component 1} + \text{Component 2} + \text{Component 3} + \text{Baseline Flow}$$

$$= 0.02426889 + 0.055415448 + 0.025313 + 0.410$$

$$= 0.514997338$$

where 0.410 is parameter \( i \) from Table A3-1

Note that normalized target release should be kept in the range of zero and one:

$$\text{if } \text{Target Release}_{normal} > 1 \text{ then } \text{Target Release}_{normal} = 1$$

$$\text{if } \text{Target Release}_{normal} < 0 \text{ then } \text{Target Release}_{normal} = 0$$

Now normalized target release should be transformed to the original cms scale as following:

$$\text{Target Release}$$

$$= \text{Target Release}_{normal} \times (\text{max allowable release} - \text{min allowable release}) + \text{min allowable release}$$

$$= 0.514997338 \times (12000 - 200) + 200$$

$$= 6276.97 \text{ cms}$$

The values for max allowable release and min allowable release for each lake in both seasons are presented in Section 2.5.4.