Let $q_b$ denote the volume bedload transport rate per unit width (sliding, rolling, saltating). It is reasonable to assume that $q_b$ increases with a measure of flow strength, such as depth-averaged flow velocity $U$ or boundary shear stress $\tau_b$.

A dimensionless Einstein bedload number $q^*$ can be defined as follows:

$$q_b^* = \frac{q_b}{\sqrt{RgDD}}$$

A common and useful approach to the quantification of bedload transport is to empirically relate $q_b^*$ with either the Shields stress $\tau^*$ or the excess of the Shields stress $\tau^*$ above some appropriately defined “critical” Shields stress $\tau_c^*$. As pointed out in the last chapter, $\tau_c^*$ can be defined appropriately so as to a) fit the data and b) provide a useful demarcation of a range below which the bedload transport rate is too low to be of interest.

The functional relation sought is thus of the form

$$q_b^* = q_b^*(\tau^*) \quad \text{or} \quad q_b^* = q_b^*(\tau^* - \tau_c^*)$$
BEDLOAD TRANSPORT RELATION OF MEYER-PETER AND MÜLLER

All the bedload relations in this chapter pertain to a flow condition known as “plane-bed” transport, i.e. transport in the absence of significant bedforms. The influence of bedforms on bedload transport rate will be considered in a later chapter.

The “mother of all modern bedload transport relations” is that due to Meyer-Peter and Müller (1948) (MPM). It takes the form

\[ q_b^* = 8(\tau^* - \tau_c^*)^{3/2}, \quad \tau_c^* = 0.047 \]

The relation was derived using flume data pertaining to well-sorted sediment in the gravel sizes.

Recently Wong (2003) and Wong and Parker (submitted) found an error in the analysis of MPM. A re-analysis of the all the data pertaining to plane-bed transport used by MPM resulted in the corrected relation

\[ q_b^* = 4.93(\tau^* - \tau_c^*)^{1.6}, \quad \tau_c^* = 0.047 \]

If the exponent of 1.5 is retained, the best-fit relation is

\[ q_b^* = 3.97(\tau^* - \tau_c^*)^{3/2}, \quad \tau_c^* = 0.0495 \]
Bedload Relation: Modified MPM

\[ q_b^* = 3.97 \left( \tau^* - \tau_c^* \right)^{1.50} \]

\[ \tau_c^* = 0.0495 \]

Data of Meyer-Peter and Muller (5.21 mm, 28.65 mm) and Gilbert (3.17 mm, 4.94 mm, 7.01 mm)
LIMITATIONS OF MPM

The “critical Shields stress” $\tau_c^*$ of either 0.047 or 0.0495 in either the original or corrected MPM relation(s) must be considered as only a matter of convenience for correlating the data. This can be demonstrated as follows.

Consider bankfull flow in a river. The bed shear stress at bankfull flow $\tau_{bbf}$ can be estimated from the depth-slope product rule of normal flow:

$$\tau_{bbf} = \rho g H_{bf} S$$

The corresponding Shields stress $\tau_{bf50}^*$ at bankfull flow is then estimated as

$$\tau_{bf50}^* = \frac{H_{bf} S}{RD_{s50}}$$

where $D_{s50}$ denotes a surface median size. For the gravel-bed rivers presented in Chapter 3, however, the average value of $\tau_{bf50}^*$ was found to be about 0.05 (next page).

According to MPM, then, these rivers can barely move sediment of the surface median size $D_{s50}$ at bankfull flow. Yet most such streams do move this size at bankfull flow, and often in significant quantities.
LIMITATIONS OF MPM contd.

There is nothing intrinsically “wrong” with MPM. In a dimensionless sense, however, the flume data used to define it correspond to the very high end of the transport events that normally occur during floods in alluvial gravel-bed streams. While the relation is important in a historical sense, it is not the best relation to use with gravel-bed streams.
A SMORGASBORD OF BEDLOAD TRANSPORT RELATIONS FOR UNIFORM SEDIMENT

Some commonly-quoted bedload transport relations with good data bases are given below.

\[
1 - \frac{1}{\sqrt{\pi}} \int_{(0.143/\tau^*)^{-2}}^{(0.143/\tau^*)^{-2}} e^{-t^2} dt = \frac{43.5q_b^*}{1 + 43.5q_b^*}
\]

Einstein (1950)

\[q_b^* = 17 \left( \frac{\tau}{\tau_c^*} \right)^3 \left( \frac{\tau_c^*}{\tau} \right) \left( \frac{\tau}{\tau_c^*} \right)^{0.5} \tau_c^* = 0.05\]

Ashida & Michiue (1972)

\[q_b^* = 18.74 \left( \frac{\tau}{\tau_c^*} \right)^3 \left( \frac{\tau_c^*}{\tau} \right)^{0.7} \tau_c^* = 0.05\]

Engelund & Fredsoe (1976)

\[q_b^* = 5.7 \left( \frac{\tau}{\tau_c^*} \right)^{n_5}, \quad \tau_c^* = 0.037 \sim 0.0455\]

Fernandez Luque & van Beek (1976)

\[q_b^* = 11.2 \left( \frac{\tau_c^*}{\tau} \right)^{4.5} \left( 1 - \frac{\tau_c^*}{\tau^*} \right), \quad \tau_c^* = 0.03\]

Parker (1979) fit to Einstein (1950)
PLOTS OF BEDLOAD TRANSPORT RELATIONS

E = Einstein
AM = Ashida-Michiue
EF = Engelund-Fredsoe
P approx E = Parker approx of Einstein
FLBSand = Fernandez Luque-van Beek, $\tau_c^* = 0.038$
FLBGrav = Fernandez Luque-van Beek, $\tau_c^* = 0.0455$
NOTES ON THE BEDLOAD TRANSPORT RELATIONS

- The bedload relation of Einstein (1950) contains no critical Shields number. This reflects his probabilistic philosophy.
- All of the relations except that of Einstein correspond to a relation of the form
  \[ q_b^* \sim (\tau^*)^{3/2} \]
  
  In the limit of high Shields number. In dimensioned form this becomes
  \[ \frac{q_b}{\sqrt{RgD D}} = K \left( \frac{u_*^2}{RgD} \right)^{3/2} \quad \text{or} \quad \frac{Rgq_b}{u_*^3} = K \]
  
  where \( K \) is a constant; for example in the case of Ashida-Michiue, \( K = 17 \). Note that in this limit the bedload transport rate becomes independent of grain size!!
- Some of the scatter between the relations is due to the face that \( \tau_c^* \) should be a function of \( \text{Re}_p \). This is reflected in the discussion of the Fernandez Luque-van Beek relation in the next slide. (Recall that \( \text{Re}_p = \sqrt{RgD D / \nu} \).)
- Some of the scatter is also due to the fact that several of the relations have been plotted well outside of the data used to derive them. For example, in data used to derive Fernandez Luque-van Beek, \( \tau^* \) never exceeded 0.11, whereas the plot extends to \( \tau^* = 1 \).
NOTES ON THE RELATION OF FERNANDEZ LUQUE AND VAN BEEK

In the experiments on which the relation is based;

a) Streamwise bed slope angle $\alpha$ varied from near 0 to 22°.

b) The material tested included five grain sizes and three specific gravities, as given below; also listed are the values of $Re_p$ and the critical Shields number $\tau_{co}^*$ determined empirically at near vanishing bed slope angle.

<table>
<thead>
<tr>
<th>Material</th>
<th>D mm</th>
<th>R</th>
<th>$Re_p$</th>
<th>$\tau_{co}^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Walnut shells</td>
<td>1.5</td>
<td>0.34</td>
<td>106</td>
<td>0.038</td>
</tr>
<tr>
<td>Sand1</td>
<td>0.9</td>
<td>1.64</td>
<td>108</td>
<td>0.038</td>
</tr>
<tr>
<td>Sand2</td>
<td>1.8</td>
<td>1.64</td>
<td>306</td>
<td>0.037</td>
</tr>
<tr>
<td>Gravel</td>
<td>3.3</td>
<td>1.64</td>
<td>760</td>
<td>0.0455</td>
</tr>
<tr>
<td>Magnetite</td>
<td>1.8</td>
<td>3.58</td>
<td>453</td>
<td>0.042</td>
</tr>
</tbody>
</table>

c) It is thus possible to check the effect of $Re_p$ and $\alpha$ on the transport relation of Fernandez Luque and van Beek (FLvB).
The experimental values of $\tau_{co}^*$ generally track the modified Shields relation of Chapter 6, but are high by a factor ~ 2. This reflects the fact that they correspond to a condition of “very small” transport determined in a consistent way (see original reference).
CRITICAL SHIELDS NUMBER IN THE RELATION OF FLvB contd.

The ratio $\tau_c^*/\tau_{co}^*$ decreases with streamwise angle $\alpha$ as predicted by the relation of Chapter 6, but to obtain good agreement $\theta_r$ must be set to the rather high value of $47^\circ$ ($\mu_c = 1.07$).

\[
\frac{\tau_c^*}{\tau_{co}^*} = \cos \alpha \left(1 - \frac{\tan \alpha}{\mu_c}\right)
\]
SHEET FLOW

• For values of $\tau^* < \text{a threshold value } \tau_{\text{sheet}}^*$, bedload is localized in terms of rolling, sliding and saltating grains that exchange only with the immediate bed surface.

• When $\tau_s^* > \tau_{\text{sheet}}^*$ the bedload layer devolves into a sliding layer of grains that can be several grains thick. Sheet flows occur in unidirectional river flows as well as bidirectional flows in the surf zone.

• Values of $\tau_{\text{sheet}}^*$ have been variously estimated as 0.5 ~ 1.5. (Horikawa, 1988, Fredsoe and Diegaard, 1994, Dohmen-Jannsen, 1999; Gao, 2003). The parameter $\tau_{\text{sheet}}^*$ appears to decrease with increasing Froude number.

• Wilson (1966) has estimated the bedload transport rate in the sheet flow regime as obeying a relation of the form

$$q_b^* = K(\tau^*)^{3/2}, \quad K = 12$$

All the previously presented bedload relations except that of Einstein also devolve to a relation of the above form for large $\tau^*$, with $K$ varying between 3.97 and 18.74.
A VIEW OF SHEET FLOW TRANSPORT

Double-click on the image to run the video.

D = 0.116 mm
S = 0.035
U = 1.05 m/s
Fr = 1.85
τ_{sheet}^* = 0.51

rte-booksheetpeng.mpg: to run without relinking, download to same folder as PowerPoint presentations.

Video clip courtesy P. Gao and A. Abrahams; Gao (2003)
MECHANISTIC DERIVATIONS OF BEDLOAD TRANSPORT RELATIONS

A number of mechanistic derivations of bedload transport relations are available. These are basically of two types, based on two forms for bedload continuity. Let $\xi_{bl}$ be the volume of sediment in bedload transport per unit area, $u_{bl}$ be the mean velocity of bedload particles, $E_{bl}$ be the volume rate of entrainment of bed particles into bedload movement (rolling, sliding or saltation, not suspension) and $L_{sbl}$ be the average step length of a bedload particle (from entrainment to deposition, usually including many saltations). The following continuity relations then hold:

$$q_b = \xi_{bl} u_{bl}, \quad q_b = E_{bl} L_{sbl}$$

In a Bagnoldian approach, separate predictors are developed for $u_{bl}$ and $\xi_{bl}$, the latter determined from the Bagnold constraint (Bagnold, 1956). Models of this type include the macroscopic models of Ashida and Michiue (1972) and Engelund and Fredsoe (1976), and the saltation models of Wiberg and Smith (1985, 1989), Sekine and Kikkawa (1992), and Nino and Garcia (1994a,b). Recently, however, Seminara et al. (2003) have shown that the Bagnold constraint is not generally correct.

In the Einsteinean approach the goal is to develop predictors for $E_{bl}$ and $L_{sbl}$. It is the former relation that is particularly difficult to achieve. Models of this type include the Einstein (1950), Tsujimoto (e.g. 1991) and Parker et al. (2003).
CALCULATIONS WITH BEDLOAD TRANSPORT RELATIONS

To perform calculations with any of the previous bedload transport relations, it is necessary to specify:

1) the submerged specific gravity $R$ of the sediment;
2) a representative grain size exposed on the bed surface, e.g. surface geometric mean size $D_{sg}$ or surface median size $D_{s50}$, to be used as the characteristic size $D$ in the relation;
3) and a value for the shear velocity of the flow $u_*$ (and thus $\tau_b$).

Once these parameters are specified, $\tau^* = (u^*)^2/(RgD)$ is computed, $q_{b*}$ is calculated from the bedload transport relation, and the volume bedload transport rate per unit width is computed as $q_b = (RgD)^{1/2}Dq_{b*}$.

The shear velocity $u_*$ is computed from the flow field using the techniques of Chapter 5. For example, in the case of normal flow satisfying the Manning-Strickler resistance relation,

$$u_*^2 = \left(\frac{k_s^{1/3}q_w^2}{\alpha_r^2} \right)^{3/10} g^{7/10} S^{7/10}$$

and

$$\tau^* = \left(\frac{k_s^{1/3}q_w^2}{\alpha_r^2 g} \right)^{3/10} \frac{S^{7/10}}{RD}$$
ALTERNATIVE DIMENSIONLESS BEDLOAD TRANSPORT

Again, the case under consideration is plane-bed bedload transport (no bedforms).

As a preliminary, define a dimensionless sediment transport rate $W^*$ as

$$W^* = \frac{q_b^*}{(\tau^*)^{3/2}} = \frac{Rgq_b}{u^3}$$

Now all previously presented bedload transport rates for uniform sediment can be rewritten in terms of $W^*$ as a function of $\tau^*$:

$$1 - \frac{1}{\sqrt{\pi}} \int_{(0.143/\tau^*)^{-2}}^{(0.143/\tau^*)^{-2}} e^{-t^2} dt = \frac{43.5(\tau^*)^{3/2} W^*}{1+43.5(\tau^*)^{3/2} W^*}$$

Einstein (1950)

$$W^* = 17 \left(1 - \frac{\tau_c^*}{\tau^*}\right) \left(1 - \frac{\tau_c^*}{\sqrt{\tau^*}}\right), \quad \tau_c^* = 0.05$$

Ashida & Michiue (1972)

$$W^* = 18.74 \left(1 - \frac{\tau_c^*}{\tau^*}\right) \left(1 - 0.7 \sqrt{\tau_c^*/\tau^*}\right), \quad \tau_c^* = 0.05$$

Engelund & Fredsoe (1976)

$$W^* = 5.7 \left(1 - \frac{\tau_c^*}{\tau^*}\right)^{1.5}, \quad \tau_c^* = 0.037 \sim 0.0455$$

Fernandez Luque & van Beek (1976)

$$W^* = 11.2 \left(1 - \frac{\tau_c^*}{\tau_s^*}\right)^{4.5}, \quad \tau_c^* = 0.03$$

Parker (1979) fit to Einstein (1950)
SURFACE-BASED BEDLOAD TRANSPORT FORMULATION FOR MIXTURES

Consider the bedload transport of a mixture of sizes. The thickness $L_a$ of the active (surface) layer of the bed with which bedload particles exchange is given by as

$$L_a = n_a D_{s90}$$

where $D_{s90}$ is the size in the surface (active) layer such that 90 percent of the material is finer, and $n_a$ is an order-one dimensionless constant (in the range 1 $\sim$ 2).

Divide the bed material into N grain size ranges, each with characteristic size $D_i$, and let $F_i$ denote the fraction of material in the surface (active) layer in the ith size range. The volume bedload transport rate per unit width of sediment in the ith grain size range is denoted as $q_{bi}$. The total volume bedload transport rate per unit width is denoted as $q_{bT}$, and the fraction of bedload in the ith grain size range is $p_{bi}$, where

$$q_{bT} = \sum_{i=1}^{N} q_{bi} , \quad p_{bi} = \frac{q_{bi}}{q_{bT}}$$

Now in analogy to $\tau^*$, $q^*$ and $W^*$, define the dimensionless grain size specific Shields number $\tau_{i}^*$, grain size specific Einstein number $q_{i}^*$ and dimensionless grain size specific bedload transport rate $W_{i}^*$ as

$$\tau_{i}^* = \frac{\tau_{b}}{\rho Rg D_i} = \frac{u_*^2}{Rg D_i} , \quad q_{i}^* = \frac{q_{bi}}{\sqrt{Rg D_i D_i F_i}} , \quad W_{i}^* = \frac{q_{bi}}{(\tau_{i}^*)^{3/2}} = \frac{Rg q_{bi}}{(u_*)^{3} F_i}$$
It is now assumed that a functional relation exists between $q_i^*$ ($W_i^*$) and $\tau_i^*$, so that

$$q_{bi}^* = \frac{q_{bi}}{\sqrt{RgD_i D_i F_i}} = f_q(\tau_i^*) \quad \text{or} \quad W_i^* = \frac{Rgq_{bi}}{(u_*)^3 F_i} = f_W(\tau_i^*)$$

The bedload transport rate of sediment in the ith grain size range is thus given as

$$q_{bi} = F_i \sqrt{RgD_i D_i} f_q(\tau_i^*) \quad \text{or} \quad q_{bi} = F_i \frac{u_3^3}{Rg} f_W(\tau_i^*)$$

According to this formulation, if the grain size range is not represented in the surface (active) layer, it will not be represented in the bedload transport.
BEDLOAD RELATION FOR MIXTURES DUE TO ASHIDA AND MICHIE (1972)

\[ q_{bi}^* = 17 \langle \epsilon_i^* - \tau_{ci}^* \rangle \sqrt[3]{\tau_i^* - \tau_{ci}^*} \]

Basic transport relation

\[
\frac{\tau_{ci}^*}{\tau_{scg}^*} = \left\{ \begin{array}{ll}
0.843 \left( \frac{D_i}{D_{sg}} \right)^{-1} & \text{for } \frac{D_i}{D_{sg}} \leq 0.4 \\
\log(19) \frac{\log \left( \frac{D_i}{D_{sg}} \right)}{\log(19) D_i} & \text{for } \frac{D_i}{D_{sg}} > 0.4
\end{array} \right.
\]

Modified version of Egiazaroff (1965) hiding function

\[ D_{sg} = 2 \overline{\psi_s} , \quad \overline{\psi_s} = \sum_{i=1}^{N} \psi_i F_i \]

Effective critical Shields stress for surface geometric mean size

\[ \tau_{scg}^* = 0.05 \]

Note: This relation has been modified slightly from the original formulation. Here the relation specifically uses surface fractions \( F_i \), and surface geometric mean size \( D_{sg} \) is specified in preference to the original arithmetic mean size \( D_{sm} = \sum D_i F_i \).
BEDLOAD RELATION FOR MIXTURES DUE TO PARKER (1990a,b)

This relation is appropriate only for the computation of gravel bedload transport rates in gravel-bed streams. In computing \( W_i^* \), \( F_i \) must be renormalized so that the sand is removed, and the remaining gravel fractions sum to unity, \( \Sigma F_i = 1 \). The method is based on surface geometric size \( D_{sg} \) and surface arithmetic standard deviation \( \sigma_s \) on the \( \psi \) scale, both computed from the renormalized fractions \( F_i \).

\[
W_i^* = 0.00218 G \phi_i
\]

\[
\phi_i = \omega \phi_{sgo} \left( \frac{D_i}{D_{sg}} \right)^{0.0951}, \quad \phi_{sgo} = \frac{\tau_{sg}^*}{\tau_{ssrg}^*}, \quad \tau_{sg}^* = \frac{u_*^2}{RgD_{sg}}, \quad \tau_{ssrg}^* = 0.0386
\]

\[
G(\phi) = \begin{cases} 
5474 \left(1 - \frac{0.853}{\phi}\right)^{4.5} & \text{for } \phi > 1.59 \\
\exp \left[4.2(\phi - 1) - 9.28(\phi - 1)^2\right] & \text{for } 1 \leq \phi \leq 1.59 \\
\phi^{14.2} & \text{for } \phi < 1
\end{cases}
\]

\[
\omega = 1 + \frac{\sigma_s}{\sigma_0(\phi_{sgo})} \Phi_0(\phi_{sgo}) - 1
\]

\[
D_{sg} = 2 \bar{\psi}_s, \quad \bar{\psi}_s = \sum_{i=1}^{N} \psi_i F_i, \quad \sigma_s^2 = \sum_{i=1}^{N} \left(\psi_i - \bar{\psi}_s\right)^2 F_i
\]

In the above \( \omega_0 \) and \( \sigma_0 \) are set functions of \( \phi_{sgo} \) specified in the next slide.
BEDLOAD RELATION FOR MIXTURES DUE TO PARKER (1990a,b) contd.

It is not necessary to use the above chart. The calculations can be performed using the Visual Basic programs in RTe-bookAcronym1.xls
BEDLOAD RELATION FOR MIXTURES DUE TO PARKER (1990a,b) contd.
An example of the renormalization to remove sand is given below.

<table>
<thead>
<tr>
<th>Range no</th>
<th>Range mm</th>
<th>D_i mm</th>
<th>F_i</th>
<th>Range no</th>
<th>Range mm</th>
<th>D_i mm</th>
<th>F_i</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.25 ~ 0.5</td>
<td>0.35</td>
<td>0.05</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>0.5 ~ 1</td>
<td>0.71</td>
<td>0.14</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>1 ~ 2</td>
<td>1.41</td>
<td>0.07</td>
<td></td>
<td></td>
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</tr>
<tr>
<td>4</td>
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<td>0.03</td>
<td></td>
<td>1</td>
<td>2 ~ 4</td>
<td>2.83</td>
</tr>
<tr>
<td>5</td>
<td>4 ~ 8</td>
<td>5.66</td>
<td>0.06</td>
<td></td>
<td>2</td>
<td>4 ~ 8</td>
<td>5.66</td>
</tr>
<tr>
<td>6</td>
<td>8 ~ 16</td>
<td>11.31</td>
<td>0.12</td>
<td></td>
<td>3</td>
<td>8 ~ 16</td>
<td>11.31</td>
</tr>
<tr>
<td>7</td>
<td>16 ~ 32</td>
<td>22.63</td>
<td>0.25</td>
<td></td>
<td>4</td>
<td>16 ~ 32</td>
<td>22.63</td>
</tr>
<tr>
<td>8</td>
<td>32 ~ 64</td>
<td>45.25</td>
<td>0.18</td>
<td></td>
<td>5</td>
<td>32 ~ 64</td>
<td>45.25</td>
</tr>
<tr>
<td>9</td>
<td>64 ~ 128</td>
<td>90.51</td>
<td>0.07</td>
<td></td>
<td>6</td>
<td>64 ~ 128</td>
<td>90.51</td>
</tr>
<tr>
<td>10</td>
<td>128 ~ 256</td>
<td>181.02</td>
<td>0.03</td>
<td></td>
<td>7</td>
<td>128 ~ 256</td>
<td>181.02</td>
</tr>
</tbody>
</table>

Fraction sand 0.26
Fraction gravel 0.74

Sum 1 Sum 1
The Microsoft Excel workbook RTe-bookAcronym1.xls is an example of a workbook in this e-book that uses code written in Visual Basic for Applications (VBA). VBA is built directly into Excel, so that anyone who has Excel (versions 2000 or later) can execute the code directly from the relevant worksheet in the workbook RTe-bookAcronym1.xls. The relevant code is contained in three modules, Module1, Module2 and Module3 of the workbook, which may be accessed from the Visual Basic Editor.

All the code in this e-book is written in VBA in Excel modules. A self-teaching tutorial in VBA is contained in the workbook RTe-bookIntroVBA.xls. Going though the tutorial will not only help the reader understand the material in this e-book, but also allow the reader to write, execute and distribute similar code.

To take the tutorial, open RTe-bookIntroVBA.xls and follow the instructions.
NOTES ON Rte-bookAcronym1.xls

The workbook RTe-bookAcronym1.xls provides interfaces for three different implementations.

In the implementation of “Acronym1” the user inputs the specific gravity of the sediment R+1, the shear velocity of the flow and the grain size distribution of the bed material. The code computes the magnitude and size distribution of the bedload transport.

In the implementation of “Acronym1_R” the user inputs the specific gravity of the sediment R+1, the flow discharge Q, the bed slope S, the channel width B, the parameter $n_k$ relating the roughness height $k_s$ to the surface size $D_{s90}$ and the grain size distribution of the bed material. The code computes the shear velocity using a Manning-Strickler resistance formulation and the assumption of normal flow, and then computes the magnitude and size distribution of the bedload transport.

The implementation of “Acronym1_D” uses the same formulation as “Acronym1_R”, but allows specification of a flow duration curve so that average annual gravel transport rate and grain size distribution can be computed.
EXAMPLE: INTERFACE FOR “Acronym1” IN Rte-bookAcronym1.xls

**INPUT TO ACRONYM1**

<table>
<thead>
<tr>
<th>A</th>
<th>B</th>
<th>C</th>
<th>D</th>
<th>E</th>
<th>F</th>
<th>G</th>
<th>H</th>
<th>I</th>
<th>J</th>
<th>K</th>
<th>L</th>
<th>M</th>
<th>N</th>
<th>O</th>
</tr>
</thead>
<tbody>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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<td></td>
<td></td>
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<tr>
<td>2</td>
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<tr>
<td>3</td>
<td></td>
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<td></td>
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</tr>
<tr>
<td>4</td>
<td><em>Put in</em> N+1</td>
<td>8</td>
<td>Number of grain sizes specifying the surface material distribution ($\leq 21$) and Type in a value of 1 for uniform material.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
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</tr>
<tr>
<td>11</td>
<td>D (mm)</td>
<td>% finer</td>
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<tr>
<td>12</td>
<td>32</td>
<td>100</td>
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<tr>
<td>13</td>
<td>16</td>
<td>50</td>
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<td>14</td>
<td>8</td>
<td>0</td>
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</tr>
</tbody>
</table>

List each grain size in mm and percent finer in the surface grain size distribution.

- Sand must be excluded from the surface grain size distribution, so that there is no content below 2 mm.
- Grain sizes must be in descending order, and percent finer must range from 100 to 0.

**OUTPUT FROM ACRONYM1**

- $\rho_{bt}$ 1.497E-07 m$^3$/s Volume bedload transport rate per unit width
- $\phi^*$ 3.861E-02 Shields number based on surface geometric mean size

**Grain size distributions of surface and bedload**

<table>
<thead>
<tr>
<th>D (mm)</th>
<th>Surface</th>
<th>Bedload</th>
</tr>
</thead>
<tbody>
<tr>
<td>32.00</td>
<td>100.00</td>
<td>100.00</td>
</tr>
<tr>
<td>16.00</td>
<td>50.00</td>
<td>71.78</td>
</tr>
<tr>
<td>8.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

**Statistics**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Surface</th>
<th>Bedload</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_50$ mm</td>
<td>16.00</td>
<td>13.78</td>
</tr>
<tr>
<td>$\phi$</td>
<td>1.41</td>
<td>1.37</td>
</tr>
<tr>
<td>$D_10$</td>
<td>27.86</td>
<td>25.03</td>
</tr>
<tr>
<td>$D_90$</td>
<td>21.11</td>
<td>15.73</td>
</tr>
<tr>
<td>$D_50$</td>
<td>18.00</td>
<td>12.97</td>
</tr>
<tr>
<td>$D_90$</td>
<td>12.13</td>
<td>10.80</td>
</tr>
</tbody>
</table>
The sand is not excluded in the fractions $F_i$ used to compute $W_i^*$. The method is based on the surface geometric mean size $D_{sg}$ and fraction sand in the surface layer $F_s$.

$$W_i^* = G \phi_i$$

$$G = \begin{cases} 
0.002 \phi^{7.5} & \text{for } \phi < 1.35 \\
14 \left(1 - \frac{0.894}{\phi^{0.5}}\right)^{4.5} & \text{for } \phi \geq 1.35
\end{cases}$$

$$\phi_i = \frac{\tau_{sg}^*}{\tau_{ssrg}^*} \left(\frac{D_i}{D_{sg}}\right)^{-b}$$

$$\tau_{ssrg}^* = 0.021 + 0.015 \exp(-20F_s)$$

$$b = \frac{0.67}{1 + \exp(1.5 - D_i/D_{sg})}$$

$$\tau_{sg}^* = \frac{u_*^2}{RgD_{sg}}$$
EFFECT OF SAND CONTENT IN THE SURFACE LAYER ON GRAVEL MOBILITY IN A GRAVEL-BED STREAM

Wilcock and Crowe (2003) have shown that increasing sand content in the bed surface layer of a gravel-bed stream renders the surface gravel more mobile. This effect is captured in their relationship between the reference Shields number for the surface geometric mean size $\tau_{ssrg}^*$ (a surrogate for a critical Shields number) and the fraction sand $F_s$ in the surface layer:

$$\tau_{ssrg}^* = 0.021 + 0.015 \exp(-20F_s)$$

Note how $\tau_{ssrg}^*$ decreases as $F_s$ increases. The surface layers of gravel-bed streams rarely contain more than 30% sand; beyond this point the gravel tends to be buried under pure sand.
BEDLOAD RELATION FOR MIXTURES DUE TO POWELL, REID AND LARONNE (2001)

The sand is excluded in the fractions $F_i$ used to compute $W_i^*$. The method is based on the surface median size $D_{s50}$, computed after excluding sand.

\[
W_i^* = 11.2 \left( 1 - \frac{1}{\phi} \right)^{4.5}
\]

\[
\phi = \frac{\tau_i^*}{\tau_{sci}^*}
\]

\[
\frac{\tau_{sci}^*}{\tau_{sc50}^*} = \left( \frac{D_i}{D_{s50}} \right)^{-0.74}, \quad \tau_{sc50}^* = 0.03
\]

More information about bedload transport relations for mixtures can be found in Parker (in press; downloadable from http://www.ce.umn.edu/~parker/).
SAMPLE CALCULATIONS OF BEDLOAD TRANSPORT RATE OF MIXTURES

Assumed grain size distribution of the bed surface: \( D_{sg} = \) surface geometric mean, \( \sigma_{sg} = \) surface geometric standard deviation, \( F_s = \) fraction sand in surface layer.

<table>
<thead>
<tr>
<th>( \psi_i )</th>
<th>( D_{b,i}, \text{mm} )</th>
<th>( D_i, \text{mm} )</th>
<th>% Finer</th>
<th>( F_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>7.5</td>
<td>256</td>
<td>100</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>6.5</td>
<td>128</td>
<td>97</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>5.5</td>
<td>64</td>
<td>71</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>4.5</td>
<td>32</td>
<td>45</td>
<td>0.26</td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td>16</td>
<td>26</td>
<td>0.19</td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td>8</td>
<td>20</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>1.5</td>
<td>4</td>
<td>18</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>2</td>
<td>16</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>1</td>
<td>7</td>
<td>0.09</td>
<td></td>
</tr>
<tr>
<td>-0.5</td>
<td>0</td>
<td>7</td>
<td>0.05</td>
<td></td>
</tr>
<tr>
<td>-1.5</td>
<td>0.5</td>
<td>2</td>
<td>0.02</td>
<td></td>
</tr>
<tr>
<td>-1.5</td>
<td>0.25</td>
<td>0</td>
<td></td>
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</tr>
</tbody>
</table>

\[ D_{sg} = 22.3 \text{ mm}, \ \sigma_{sg} = 4.93, \ F_s = 0.16 \]

\[ D_{sg} = 40.7 \text{ mm}, \ \sigma_{sg} = 2.36 \]  
(sand excluded)
Grain Size Distributions for Bedload Calculations

- Surface
- Surface Gravel
SAMPLE CALCULATIONS, MIXTURES contd.

Other input parameters:

- \( R = 1.65 \)
- \( u_* = 0.15 \) to 0.40 m/s

Relations used:

- \( A-M = \) Ashida and Michiue (1972), sand not excluded
- \( P = \) Parker (1990), sand excluded
- \( P-R-L = \) Powell et al. (2001), sand excluded
- \( W-C = \) Wilcock and Crowe (2003), sand not excluded

Output parameters

- \( q_G = \) volume gravel bedload transport rate per unit width
- \( D_{Gg} = \) geometric mean size of the gravel portion of the transport
- \( \sigma_{Gg} = \) geometric standard deviation of the gravel portion of the bedload
- \( p_G = \) fraction gravel in the bedload transport (only for A-M and W-C):
  - fraction sand = 1 - \( p_G \)
SAMPLE CALCULATIONS, MIXTURES contd.

Gravel Bedload Transport Rate

\[ q_G (m^2/s) \]

\[ u^* (m/s) \]
SAMPLE CALCULATIONS, MIXTURES contd.

Geometric Mean Size of Gravel Bedload

![Graph showing geometric mean size of gravel bedload vs. $u^*$ (m/s)]
SAMPLE CALCULATIONS, MIXTURES contd.

Geometric Standard Deviation of Gravel Bedload

\[ \sigma_{Gg} \]

\[ u^* \text{ (m/s)} \]
SAMPLE CALCULATIONS, MIXTURES contd.

Fraction Gravel in Bedload

\[ \rho_G \] vs. \( u^* \) (m/s)

- A-M
- W-C

0.5
0.1
0
1
1
0.1
u^* (m/s)
COMPUTATIONS OF ANNUAL BEDLOAD YIELD

It is necessary to have a flow duration curve to perform the calculation. The flow duration curve specifies the fraction of time a given water discharge is exceeded, as a function of water discharge.

This curve is divided into M bins \( k = 1 \) to M, such that \( p_{Q_k} \) specifies the fraction of time the flow is in range \( k \) with characteristic discharge \( Q_k \). The value \( u_{*k} \) must be computed for each range. For example, in the case of normal flow with constant width \( B \), the Manning-Strickler resistance relation from Chapter 5 yields

\[
u_{*k}^2 = \left( \frac{k_s^{1/3}Q_k^2}{B^2\alpha_r^2} \right)^{3/10} g^{7/10} S^{7/10}
\]

The grain size distribution of the bed material must be specified; \( k_s \) can be computed as 2 \( D_{s90} \) (for a plane bed). Once \( u_{*k} \) is computed, either \( q_{b,k} \) (material approximated as uniform) or \( q_{b_i,k} \) (mixtures) is computed for each flow range, and the annual sediment yield \( q_{ba} \) or total yield \( q_{bTa} \) and grain size fractions of the yield \( p_{ai} \) are given as

\[
q_{ba} = \sum_{k=1}^{M} q_{b,k} p_{Q_k} \quad \text{or} \quad q_{bTa} = \sum_{i=1}^{N} \left( \sum_{k=1}^{M} q_{b_i,k} p_{Q_k} \right) , \quad p_{ai} = \frac{\sum_{k=1}^{M} q_{b_i,k} p_{Q_k}}{\sum_{i=1}^{N} \left( \sum_{k=1}^{M} q_{b_i,k} p_{Q_k} \right)}
\]

Expect the flood flows to contribute disproportionately to the annual sediment yield. An implementation is given in “Acronym1_D” of Rte-bookAcronym1.xls.
REFERENCES FOR CHAPTER 7


Horikawa, K., 1988, Nearshore Dynamics and Coastal Processes, University of Tokyo Press, 522 p.
REFERENCES FOR CHAPTER 7 contd.


Parker, G., 1990b, The ACRONYM Series of PASCAL Programs for Computing Bedload Transport in Gravel Rivers, *External Memorandum* M-200, St. Anthony Falls Laboratory, University of Minnesota, Minneapolis, Minnesota USA.


REFERENCES FOR CHAPTER 7 contd.


